DOINGWHATWORKS

PRESENTATION
8:56 min

Full Details and Transcript

Effective Problem-Solving Instruction

February 2012

Topic IMPROVING MATHEMATICAL PROBLEM SOLVING IN GRADES 4 THROUGH 8

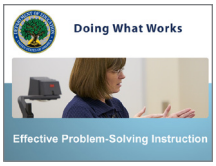
Practice PROBLEM-SOLVING INSTRUCTION

Highlights

- Teachers can use specific strategies during problem-solving instruction to build students' understanding of core mathematics concepts and skills. The three important strategies that apply at all grade levels and in all areas of mathematics are: use of visual representations, encouragement of multiple approaches to solving problems, and linking mathematical and algebraic notation to intuitive approaches.
- Using visual representations prior to introducing equations brings several benefits to students, helping them organize the information in a problem, distinguish relevant from irrelevant information, clarify the goal of the problem, see relationships, and then focus on mathematical reasoning.
- If teachers encourage use of select visual representations consistently, students will more likely apply them on their own and will rely less on drawing narrative pictures. Commonly used representations are tables, strip diagrams, percent bars, and number lines. Strip diagrams are especially useful for comparisons; tables are helpful for ratio and proportion problems.

- When teachers explicitly teach that there is more than one way to solve a problem, students learn to be more flexible in their thinking and efficient in choosing solutions. This includes demonstrating routinely two or more ways to approach a problem and comparing the problem-solving approaches as well as analyzing solutions for efficiency.
- While students may be able to solve simple problems intuitively, they will need facility with mathematical notation for more complex problems. Teachers can build up that facility by explicitly linking ideas in word problems to equations.
- To provide practice with notation, teachers can provide worked examples with related mathematical equations.
- Overall, the goal should be to help students articulate mathematically valid explanations of their reasoning.

Full Transcript



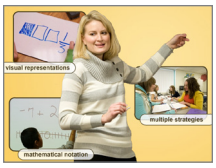
Slide 1: Welcome

Welcome to the overview on Effective Problem-Solving Instruction.

Slide 2: Opportunities in process

When teaching students how to solve mathematics problems, teachers need to be continually alert for instructional opportunities. As students share their reasoning while working through a challenging problem, observant teachers will have instructional “openings” that can be used to introduce or reinforce familiar mathematics concepts and methods.

Taking advantage of these opportunities to further students’ skills and understanding is at the core of effective teaching.



Slide 3: Three instructional strategies

There are three major strategies that teachers can use to guide students as they learn how to tackle mathematics problems.

- Teach students how to create visual representations of relevant information in a problem,
- Encourage students to explore multiple ways to solve a problem, and
- Demonstrate how to break a problem into steps that can then be expressed through formal mathematical notation.

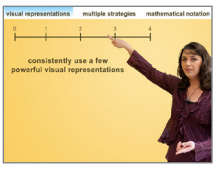
Let's look at these one at a time.

Slide 4: Visual representations

Research suggests that students who develop visual representations prior to working with equations are more effective problem solvers. This may be because visual representations help students develop a deeper understanding of the problems they are working with.

The right type of representation can help a student get a coherent view of the problem by identifying and organizing pieces of relevant mathematical information. Specifically, the visualization helps students summarize what key information is known and see what the problem is asking them to solve for.

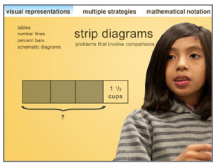
Use of an appropriate visual can also reveal the relationships between the quantities identified in the problem. Once students grasp these relationships, they can focus their attention on mathematics reasoning and the problem-solving process. Students are also in a better position to express a problem using equations.



Slide 5: Familiarity with powerful visuals

Teachers are advised to consistently use a few powerful visual representations. A powerful model or representation is one that has a variety of applications, such as a number line or strip diagram.

If students work with a particular visual representation when they encounter a certain type of problem, they are more likely to grow comfortable with that tool and use it on their own. Students also are then less likely to use narrative pictures, which can distract them from the essential mathematical information in the problem.



Slide 6: Types of visuals

Tables, number lines, strip diagrams, percent bars, and schematic diagrams are among the most frequently used visuals. Schematic diagrams use abstract graphic symbols rather than realistic pictures and include only relevant problem elements. Of course, some visuals are better suited for particular types of problems.

For example, strip diagrams work well for problems that involve comparisons. In this example, Cheri is using a strip diagram to find out how much chili was served for dinner when the family has consumed 3/4 of the chili and there is 1 1/2 cups left.

Tables work well for ratio and proportion problems. Here, Pedro and Sally have drawn a ratio table to figure out how much paint of each color is needed to paint five classrooms if one gallon of yellow and five gallons of beige are required to paint two classrooms.

Slide 7: Talking through the visual

To help students learn how to employ visual representations, teachers can talk aloud about what they are thinking and the decisions they are making as they reason through a problem.

During the thinkaloud, teachers should demonstrate how they identify what information will be placed in the diagram and what aspects of the problem are irrelevant.

It is essential that teachers explain why the representation they are using is appropriate for the problem at hand.

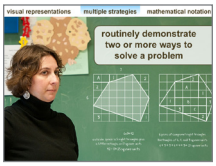
It is just as important for students to explain to the teacher how they are setting up a diagram and representing the quantities in the problem.

By listening to their students' reasoning, teachers are better able to identify and address possible misconceptions. As teachers grow to understand how their students are thinking about problems, they will be able to introduce them to the idea that there is more than one way to think about a problem.

Slide 8: Multiple approaches

Researchers recommend that students be taught explicitly that problems can be solved in more than one way.

Students who practice multiple strategies and share their solutions become more flexible and efficient in problem solving, and are more likely to see options when approaching a problem.

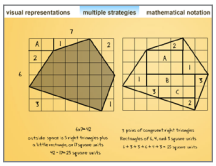


Slide 9: Demonstrating solutions

Teachers should routinely demonstrate two or more ways to solve a problem.

Looking at worked examples with multiple solutions side by side gives students practice comparing similarities and differences in the strategies, which can help strengthen analytical thinking.

Students also benefit by being expected to use multiple methods to solve problems themselves.



Slide 10: Ease and efficiency

As students get used to using multiple approaches, teachers should talk through the reasons why one solution might be favored over another. This will help students understand that strategies should be chosen based on ease and efficiency.

It can also be helpful for a teacher to demonstrate approaches to problems that are not successful and discuss why they seem like they would work, but why they don't.

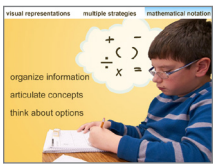
Slide 11: Culture of problem solving

When teachers routinely focus on students' thinking and reasoning, and not merely on the mechanics of a particular solution, students begin to expect that there will be multiple ways to approach any problem and that for some problems there will be more than one solution.

Comparing different strategies does take time, as students need to comprehend each approach before contrasting it with others.

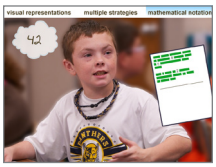
When deciding which students will share their solutions with the whole class, it is best to choose three or four students who have used different approaches to the problem.

Dividing the class into small peer groups that compare solutions and explain their approaches to each other can help students who are reluctant to discuss their reasoning in front of the whole class. As students observe each other's reasoning and solutions, they more clearly understand that there are multiple ways to approach problems and they can begin to analyze those approaches for efficiency.



Slide 12: Mathematical notation

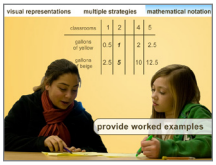
Mathematical notation helps students organize the information in a problem, articulate mathematics concepts, and think about their options for solving a problem.



Slide 13: Notation for challenging problems

Students may prefer to approach problems intuitively or informally, rather than spend the time to develop an equation. Intuitive approaches may work for simple problems, but notational tools are needed for more complex and challenging problems.

By explicitly linking the ideas in a word problem to an equation, teachers demonstrate how to express problems through mathematical notation, including algebraic notation.



Slide 14: Using worked problems

One way to help students become comfortable with mathematical notation is to provide worked examples of word problems along with related mathematical expressions or equations. A teacher can challenge students to match key information in the problem statement with the related component in the equation.

Slide 15: Opportunity for new concepts

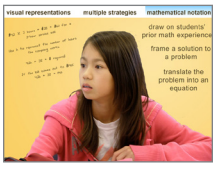
Problem solving provides a teacher with many opportunities to review or explain relevant mathematical concepts and introduce new ways of reasoning.

When preparing problems, teachers should work through several approaches in advance. This will help them anticipate the mathematical concepts students may attempt to use in order to solve a problem.

Slide 16: Mathematically valid explanations

A goal of problem solving is to help students learn to articulate mathematically valid explanations. Teachers may need to support students in organizing their ideas and rewording explanations so they are mathematically correct.

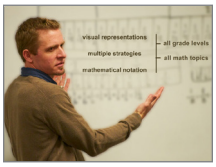
The teacher's probing questions can help students refine their thinking and develop explanations that are logical and can be generalized and applied in other problem situations.



Slide 17: Symbolic notation

Students will need time to become familiar with the abstract symbolic notation of algebra.

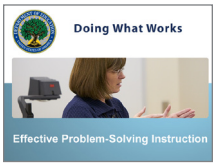
Teachers may use arithmetic problems as a first step, drawing on students' prior math experience to frame a solution to a problem before translating the same problem into an equation with variables representing the problem's components.



Slide 18: Summary

These three core strategies—visual representations, multiple approaches to problems, and mathematical notation—are the teacher's primary tools for taking advantage of the instructional opportunities available when problem solving.

All three can be applied at all grade levels and with all mathematical topics.



Slide 19: Learn more

To learn more about Effective Problem-Solving Instruction, please explore the additional resources on the Doing What Works website.

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