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Full Details and Transcript

Algebraic Notation in Problem Solving

Ken Koedinger, Ph.D., September 2011

Topic IMPROVING MATHEMATICAL PROBLEM SOLVING IN GRADES 4 THROUGH 8

Practice PROBLEM-SOLVING INSTRUCTION

Highlights

» Dr. Koedinger explains that mathematical notation is challenging and important for students to learn in problem solving.

» He illustrates through an example the use of algebraic notation, which helps prepare students to understand complex problems that are hard to solve intuitively and facilitates solutions.

» Dr. Koedinger models a technique of supplying an algebraic equation with the problem in order for students to explain the meaning of each component of that equation as it relates to the original problem.

» He recommends using problems in which equivalent expressions emerge in order to demonstrate mathematical principles that transform one expression into another form.

» Dr. Koedinger concludes that algebraic notation in problem solving is a powerful tool that takes time for students to learn.

About the Interviewee Kenneth R. Koedinger, Ph.D., is a professor of human-computer interaction and psychology at Carnegie Mellon University. His research focuses on educational technologies that dramatically increase student achievement. He works to create computer simulations of student thinking and learning that are used to guide the design of educational materials, practices, and technologies. These simulations provide the basis for an approach to educational technology called Cognitive Tutors that offers rich problem-solving environments in which students can work and receive just-in-time learning assistance, much like what human tutors offer. Dr. Koedinger has developed Cognitive Tutors for both mathematics and science and has tested the technology in laboratory and classroom environments. His research has contributed new principles and techniques for the design of educational software and has produced basic cognitive-science research results on the nature of mathematical thinking and learning. Dr. Koedinger has authored 126 peer-reviewed publications, 10 book chapters, and 59 other papers, and he has been a project investigator on 19 major grants. He is a cofounder of Carnegie Learning, Inc., and he directs the Pittsburgh Science of Learning Center (learnlab.org).

Full Transcript

00:04 I am Ken Koedinger. I am a professor of human-computer interaction and psychology at Carnegie Mellon University.

00:11 Recognizing notations and articulating them is very important as a component in problem solving. Students need to be able to use those mathematical tools to be effective problem solvers, and it turns out that those notations are quite challenging for students. One of the reasons that we know that mathematical notation is surprisingly difficult for students are studies where we have compared student problem solving in word problems to solving those exact same problems when they are given an equation. And what we have discovered is that there are many elements of learning the notational system that are surprisingly challenging.

00:55 This recommendation is really important in helping teachers and developers to get past their expert blind spots and to understand what's really hard for students and develop instruction that can then focus in on those student difficulties.

01:11 Algebraic notations are an important part of problem solving. But sometimes we can do activities that are going to facilitate problem solving by focusing on the notation side. When you are translating an English story problem into algebra, a big difficulty for students is producing that new algebraic language. The importance of algebraic notation for solving problems comes out particularly as problems get more complex.

01:42 For simpler problems, students are often able to solve simpler "algebraic problems" without using algebra at all. But that only goes so far, and we have research that shows particularly where problems involve more complex forms where the unknown is referenced more than once in the problem. Those are really hard to solve intuitively. Like, for instance, John bought a coat at a 20% discount. He paid $38. What was the original price? You can't just divide the 20% into 38 and get the answer—you will get the wrong answer—or even multiply 20% by 38. The equation there, $x - .2x = 38$, helps you understand how to solve that problem and facilitates a more effective solution to those kinds of more complex problems.

02:35 If you use simpler story problems early, which makes some sense, and students don't use the notation, they may get those answers right but you may not be preparing them for the more challenging problems. You want them to get to use those more sophisticated strategies so that when they are in more challenging situations, the powerful algebra strategy—that overhand serve—can be applied and be effective.

03:02 There are some techniques teachers can use to help students understand mathematical and algebraic notation, in particular, and make connections to problem solving. One of those is to give a student a story problem and the equation that models that story problem, and then ask them to explain components of that equation in terms of what they refer to in the story problem.

03:26 For instance, you might have a story problem about Joseph earning money selling seven CDs and his old headphones. He said the headphones were $10 and he made $40.31. How much did he sell each CD for? So the equation for that problem is 10 + 7x = $40.31. Now, you would give all of that to the student. The activity for the student now starts with explaining elements of that equation. What does the x represent? What does the 7x represent in that equation? What does the 10 + 7x represent in that equation? And each of those components can be referenced back to the equation. And in fact the $40.31, that represents the same thing—how much he made—as does the 10 + 7x. One of the tricks to these is that students sometimes think they can only give an explanation once, but they need to understand that in problems with equations there are two ways to represent the same thing, and that's why we put an equal sign between them.

04:34 It's not only that level of explanation that's important in these kinds of problems; it's also important that each little subcomponent of the expression has a meaning. And helping students pull that apart, you want them to be able to decompose the notation, make sense of the parts, and make sense of how the parts fit together to tell a story in an equation that matches the story in English.

05:02 Sometimes in class when you give a problem, you get different solutions from students—oftentimes, actually. Let's say you give a geometric pattern problem where you ask students to come up with some mathematical form. So some students might say, "Well, it's 3x + 6." Another student might say, "No, I think it's x + 2, multiplied by 3." They are actually both right; those are both good expressions. Those are great opportunities to help students think about equivalence of different algebraic expressions and even how you can transform one expression into the other.

05:44 To have those first emerge from a patterning problem or a story problem and be able to say, yeah, both students are right, but let's now talk about why those two expressions are equivalent, not only because they model the same problem but because we can apply a mathematical principle that transforms one into another.

06:05 The role of algebraic notation in problem solving is that it's a powerful tool and one of the things that we have discovered is that tool takes time to learn. We should be having students use the language of algebra as early as possible in school and repeatedly.

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