PRESENTATION

6:18 min

Full Details and Transcript

Connecting Mathematical Ideas to Notation

Sybilla Beckmann, Ph.D.

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Topic IMPROVING MATHEMATICAL PROBLEM SOLVING IN GRADES 4 THROUGH 8

Practice PROBLEM-SOLVING INSTRUCTION

Highlights

» Problem solving is an excellent vehicle for connecting mathematical ideas to mathematical notation.

» Dr. Sybilla Beckmann provides an example of a problem statement: three submarine sandwiches are shared equally among four students. How much of a sandwich does one student get? She shows how to solve the problem in several ways and how to connect those to illustrate division and fraction concepts.

» A second problem example is presented: Carla and Jessica each have some money. Carla has $11 more than Jessica. Altogether they have $85. How much does Carla have? How much does Jessica have?

» Dr. Beckmann illustrates different approaches to solving the problem: using a strip diagram to facilitate reasoning and a guess-and-check strategy.

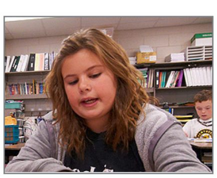
» She shows how both approaches can be used to transition to reasoning using symbolic algebra with algebraic notation.

About the Interviewee

Sybilla Beckmann is Josiah Meigs Distinguished Teaching Professor of Mathematics at the University of Georgia. She has a Ph.D. in mathematics from the University of Pennsylvania and taught at Yale University as a J.W. Gibbs Instructor of Mathematics. Dr. Beckmann has done research in arithmetic geometry, but her current main interests are the mathematical education of teachers and mathematics content for students at all levels, but especially for pre-K through the middle grades. She developed several mathematics content courses for prospective elementary school teachers at the University of Georgia and wrote a book for such courses, *Mathematics for Elementary Teachers*, published by Addison-Wesley, now in a third edition. She is interested in helping college faculty learn to teach mathematics content courses for elementary and middle grades teachers, and she works with graduate students and postdoctoral fellows toward that end. As part of this effort, Dr. Beckmann directs the Mathematicians Educating Future Teachers component of the University of Georgia Mathematics Department's VIGRE II grant.

Dr. Beckmann was a member of the writing team of the National Council of Teachers of Mathematics' *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*; was a member of the Committee on Early Childhood Mathematics of the National Research Council and co-author of its report, *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*; has worked on the development of several state mathematics standards; and was a member of the mathematics writing team for the Common Core State Standards Initiative. Several years ago Dr. Beckmann taught an average sixth-grade mathematics class every day at a local public school in order to better understand school mathematics teaching.

Full Transcript



Slide 1: Welcome

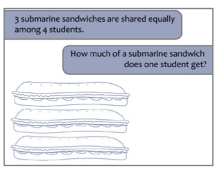
Welcome to Connecting Mathematical Ideas to Notation.

Slide 2: Introducing Sybilla Beckmann

I am Sybilla Beckmann, Josiah Meigs Distinguished Teaching Professor at the University of Georgia. I was a member of the panel for the Problem Solving guide as well as a member of the panel for the Response to Intervention guide.

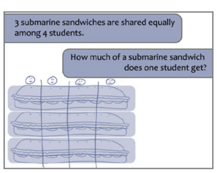
Slide 3: Problem solving

Problem solving can be an excellent vehicle for connecting mathematical ideas to mathematical notation. Let me illustrate that with this example: Three submarine sandwiches are shared equally among four students. How much of a submarine sandwich does one student get?



Slide 4: Submarine sandwich example

Students might solve this problem by drawing three submarine sandwiches. If they are going to share those sandwiches equally among four friends, they might cut each sandwich up into four equal parts and then they might give one person one part from each of the three sandwiches. So at this point students might realize that, oh, each person will get three parts, each of which is one-fourth of the submarine sandwich.

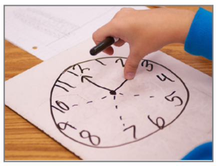


Slide 5: Expressing two mathematical concepts

This means that the three subs that have been divided equally among four shares, which we can think of as three divided by four, can also be expressed as three parts, each of which is one-fourth of a sub and that is the fraction three-fourths—that those two things are equal. So we have two pieces of notation and two mathematical ideas. One is division, three divided by four. And the other is fractions; in this case, we have three parts, each of which is a fourth, that's the fraction three-fourths. And those two things are equal.

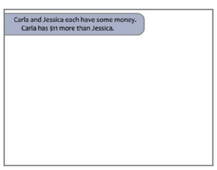
Slide 6: The connection between division and fractions

The problem-solving situation is an opportunity to develop a real mathematical idea, and that is a connection between division and fractions. A problem-solving situation can bring such an idea to the fore and allow for the notation and mathematical ideas to be discussed in the classroom.



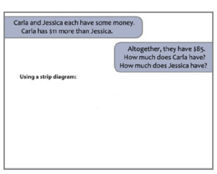
Slide 7: Extending students’ thinking

Problem solving can be an excellent venue for comparing different ways of thinking about a situation, and it can be used to extend students’ thinking. It can help students go from something that they understand and know and can state in their own words to bring them perhaps to the cusp of some new knowledge that they are ready to start to develop, but perhaps haven’t quite yet appreciated.



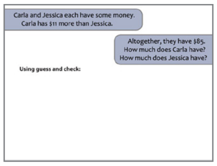
Slide 8: Problem: Jessica’s and Carla’s money

Let’s take a look at this problem: Carla and Jessica each have some money. Carla has $11 more than Jessica, altogether they have $85. How much does Carla have? How much does Jessica have?



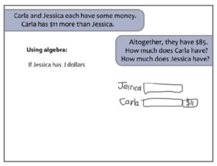
Slide 9: Solving with strip diagrams

One student might solve this problem using a strip diagram. The student draws two strips that stand for the amount of money that Jessica has and then the amount of money that Carla has, also is that same amount plus another $11. And we see how together it is $85. And the student might then reason that if they take away that $11 from the $85, then the remaining amount, which is $74, must be split equally among those two parts that are the same size. So the student then divides 74 by 2 and gets 37, and therefore Jessica must have had $37 and then Carla will have $11 more. And the student also checks the work that, yes, indeed, that does fit and those do add up to $85.



Slide 10: Guess and check

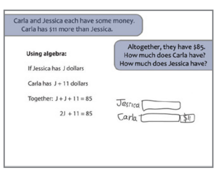
Now another student might use a guess-and-check strategy and just try some numbers for Jessica and make Carla's $11 more and then check what the total is in each case. Both of those ways of reasoning lead to a correct answer, and both of those ways of reasoning can be used to discuss another strategy, which is to use algebra.



Slide 11: Using symbolic algebra

If we think of Jessica's strip there, that rectangle, as standing for J dollars, then we also see that Carla's strip stands for J + $11. If we add all of those parts together, we will have J + J + another 11, and that's supposed to equal $85.

Now, not only that, not only do we get the equation from the strip diagram, but also the way of reasoning with the strip diagram actually parallels algebraic reasoning.

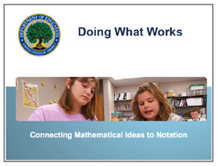


Slide 12: Solving with equations

If we look at this equation 2J + 11 = 85, the next step might be subtract 11 from both sides. And we see that reflected in the strip diagram by seeing that we have those 2J pieces or those 2 rectangular pieces that are unlabelled after we subtract $11 from the $85, 2J = 74, and we now divide both sides by 2, is also reflected in the strip diagram. The reasoning parallels, the algebraic reasoning parallels what is being done with the strip diagram, and therefore we could use the strip diagram as a transition point into that more symbolic algebra.

Slide 13: More advance reasoning

Problems can be an opportunity to take that next step to go into the more advanced way of reasoning and to extend students' thinking beyond what they are already comfortable with and to teach them a new way of thinking about something.



Slide 14: Learn more

To learn more about Connecting Mathematical Ideas to Notation, please explore the additional resources on the Doing What Works website.

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