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Number Lines: A Key Representational Tool

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Topic DEVELOPING EFFECTIVE FRACTIONS INSTRUCTION FOR K-8

Practice FRACTIONS AS NUMBERS

Highlights

- Dr. Okamoto beings with the types of misconceptions that children have about fractions, especially those related to not understanding fractions as numbers.
- She explains that fractions are numbers that expand the number system by adding more precise units of measure.
- She demonstrates with examples showing why part-whole area models can be confusing for some children, and she shows problems that may also confuse children into thinking that fractions are made up of whole numbers.
- Dr. Okamoto makes a strong recommendation for using number lines as a robust representational model and demonstrates how to use the number line for adding fractions.
- She models measurement activities with strips of paper of different lengths.
- She also demonstrates concepts of fraction density, using parallel number lines portioned into different fractional parts to help children learn equivalents.

About the Interviewee

Yukari Okamoto, Ph.D., is a professor in the Department of Education at the University of California, Santa Barbara. Her work focuses on cognitive development, the teaching and learning of mathematics and science, and cross-cultural studies. She is particularly interested in children's acquisition of mathematical, scientific, and spatial concepts, and she participated in the video studies of mathematics and science teaching as part of the Third International Mathematics and Science Study (TIMSS). Dr. Okamoto's recent publications include *Fourth Graders' Linking of Rational Number Representation: A Mixed Method Approach* and *Comparing U.S. and Japanese Elementary School Teachers' Facility for Linking Rational Number Representations*.

Full Transcript

00:00 I am Yukari Okamoto, professor of education at the University of California, Santa Barbara. I was a member of the fractions Practice Guide panel.

00:14 One of the problems students have when they are first introduced to fractions is that they think a fraction is made up of two whole numbers. For example, when asked to add two fractions like, say, two-thirds and one-fifth, some students add the numerators together, then the denominators, and then come up with three-eighths. Another example of a common misconception is the students think three-eighths is bigger than three-fifths because eight is bigger than five. So why do students have this kind of misconception? Part of the problem is that there is little emphasis on the development of rational number and understanding in early grades. Fractions are numbers that can be used to measure quantities just like whole numbers. Fractions expand the number system beyond whole numbers. And fractions also provide more precise units of measure.

01:10 It seems like a simple idea to adults, but it's not for young students. So when teaching fractions, we often see a pie or area model to convey fraction concepts. The idea is that a fraction represents a part or parts of a whole. In this example, we see a pizza cut into three equal pieces, with two of them shaded or faded away. So if you

ate two pieces, then you ate two-thirds of the pizza. This explanation may make sense to some students but not to all. So what happens is that the students count each of the three pieces as if they are whole numbers. So they don't necessarily realize that each part is one-third of a unit, and together they make the unit of one.

01:59 Here's a more problematic example. Here's a picture of six cookies of different shapes and flavors. One is a chocolate cookie, and the rest are raisin cookies. The task is to write a fraction for the relation of the chocolate cookie to all cookies. To me, it's problematic that all the students need to do to succeed is to count the number of cookies and put the resulting whole numbers in the blanks above and below the division marker. I am personally concerned that this type of instruction may lead to students' misconception that fractions are made up of whole numbers.

02:40 Fractions themselves are numbers with magnitudes that extend the number system. It's not easy to see this in this example, and this example also doesn't give you a good sense of the unit of measure. Is it the single cookie or the entire set of cookies? It's not clear.

03:00 In this example, we want to show students what it means to add two fractions, one-third and one-third, using the part-whole approach. We can first shade one-third and shade another third and then come up with the answer, two-thirds. This is a good way to use a part-whole approach. But as you can see below, students could interpret this situation incorrectly. They might add these two, and they might add the numerators together and the denominators together and come up with two-sixths.

03:36 How can we help student develop the idea that fractions are numbers? We, the Fractions Panel, strongly recommend that teachers use number lines as a representational tool help students understand important fraction concepts.

03:53 I think measurement activities are particularly useful for this purpose. Here, I have some strips of paper. Using something like

this, or fraction strips, we can find out how many strips of paper we need to measure different things.

04:07 Here is a pencil. Let's measure this using these paper strips. And this pencil is longer than one strip but shorter than two strips. How can we express this extra amount? The class can discuss various methods for doing this. Folding this strip like this works in this case. Then the teacher can talk about the notion of one and a half strips. When doing this sort of activity, it's important that teachers use strips of different length representing different units of measure. Ask the students to measure an object using different strips. The length of the object could be expressed differently, say, one-and-a-half blue strips in this case, or if you are using red strips of different length, it could be three red strips long.

04:58 This type of activity should help students become aware of the importance of measurement units. The number line is also useful in helping students understand what it means to add fractions. The addition of fractions like 1/3 + 1/3 could be introduced like this, using the number line. Here, the number line is partitioned into three parts. So the first one-third is marked here, and then you add another one-third to come up with the answer, two-thirds.

05:31 Here, parallel number lines are partitioned into different fractional parts. Something like this can also help students learn equivalence of fractions. Using a straight edge like a pencil, students will find out one-half is the same as two-fourths and also same as six-twelfths. Or you line it up like this, and students will find out one-quarter is the same as three-twelfths. Again, you can line up like this and see that three-fourths is the same as nine-twelfths.

06:09 By partitioning a number line repeatedly, students realize that there are an infinite number of numbers between any two adjacent whole numbers. This is a very important notion called *fraction density*. Number lines are also useful in helping students translate among various notations of fractions such as decimals and percents. Number lines convey important properties of fractions that other methods, such as the part-whole approach, do not.