DOINGWHATWORKS

PRESENTATION
6:09 min

Full Details and Transcript

Making Sense of Computational Procedures

May 2011

Topic DEVELOPING EFFECTIVE FRACTIONS INSTRUCTION FOR K-8

Practice OPERATIONS WITH FRACTIONS

Highlights

- In the intermediate grades students should master addition and subtraction of fractions and decimals. In the middle grades, they should become proficient in multiplying and dividing fractions and decimals.
- Teaching fractions by connecting conceptual understanding with procedural fluency will strengthen student learning.
- Teachers can use different types of visual representations and manipulatives to help students gain insight into why computational procedures work.
- Students can strengthen their reasoning about fractions when they use estimation to predict or judge the reasonableness of an answer to a problem.
- The most common misconceptions involve treating fractions as if they were whole numbers or misapplying a procedure from a different fraction operation.

- Not recognizing that different denominators indicate different-sized unit fractions leads to many common misconceptions, such as adding only numerators in an equation.
- Students quite often have difficulty with the "invert and multiply" procedure for fraction division. The problem arises when students are taught this procedure without an understanding of why it works.
- Use students' mistakes in computations with fractions as a window into students' thinking.

Full Transcript

Slide 1: Welcome

Welcome to the overview on Making Sense of Computational Procedures.

Slide 2: Benchmarks

There is a growing consensus about the benchmarks that elementary and middle school students must achieve in order to be proficient in mathematics and ready for algebra. Several of those benchmarks involve computation with fractions.

In the intermediate grades students should master addition and subtraction of fractions and decimals. In the middle grades, they should become proficient in multiplying and dividing fractions and decimals.

Slide 3: Conceptual with procedural

Research shows that when students understand why procedures work, they are more likely to become fluent in computation. Teaching fractions by connecting conceptual understanding with procedural fluency will strengthen student learning.

Slide 4: Ways to develop concepts

Conceptual understanding of fraction operations can be developed by:

- Using visual representations to demonstrate concepts,
- Presenting problems in real-world situations,
- Giving students opportunities to use estimations and make reasonable predictions, and
- Directly addressing common misconceptions.

Slide 5: Representations: Fraction circles and addition

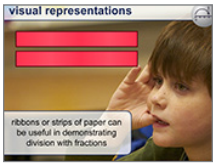
Teachers can use different types of visual representations and manipulatives to help students gain insight into why computational procedures work. For example, area models like fraction circles can help students see the need for common denominators when adding fractions.

Here, we see a visual representation that shows addition of fractions with different denominators by identifying a common denominator and translating the problem into equivalent fractions.

Slide 6: Pictorial representation and multiplication

Pictorial representations can help students grasp the concept of multiplying fractions.

In this example, the challenge is to decide how much of a cake can be frosted with one-fourth of a cup of icing if one cup of icing covers only two-thirds of the cake. A pictorial representation allows students to partition the cake from one whole, to two-thirds, to one-fourth of two-thirds. In short, to find a fraction of a fraction.



Slide 7: Fraction strips and division

Simple ribbons or strips of paper can be useful in demonstrating division with fractions.

In this problem, students cut same-length ribbons into fourths and halves to find what is one-half divided by one-fourth. Two-fourths fit into one-half of the ribbon…so one-half divided by one-fourth is two.

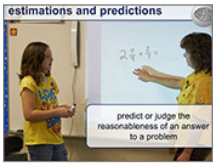
Slide 8: Number lines

Number lines are the representational tool of choice for many educators because they offer flexibility with all types of fractions, including improper fractions and negative fractions.

The overview on Recognizing Fractions as Numbers includes more information about number lines.

Slide 9: Real-world contexts

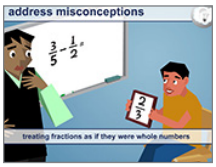
Problems such as frosting a cake or measuring and cutting ribbons help students see how computation is used in real-life situations and allows them to build on their own intuitive understanding of fractions.



Slide 10: Estimations and predictions

Students can strengthen their reasoning about fractions when they use estimation to predict or judge the reasonableness of an answer to a problem.

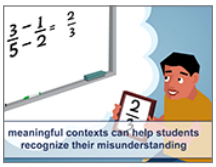
Teachers can provide opportunities and discuss strategies for students to make estimates and compare those estimates to their completed solutions. Estimations help students improve their reasoning and make more accurate predictions.



Slide 11: Common misconceptions

Often students hold misconceptions about fractions that get in the way of understanding or learning computational procedures.

The most common misconceptions involve treating fractions as if they were whole numbers or misapplying a procedure from a different fraction operation.



Slide 12: Meaning of numerators and denominators

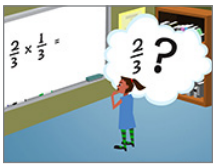
A common mistake students make when adding or subtracting fractions is to add or subtract the numerators and denominators separately.

Meaningful contexts can help students recognize their misunderstanding. If a student has a piece of wood that is three-fourths of a foot long and cuts off a piece that is half a foot long, but ends up with one foot of board, he can clearly see that he's made a mistake and needs to rethink his process.

Slide 13: Mistakes with denominators

Adding and subtracting fractions requires a common unit fraction. Not recognizing that different denominators indicate different-sized unit fractions leads to many common misconceptions, such as adding only numerators in an equation.

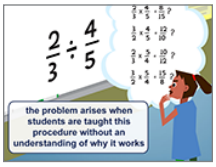
Use of fraction strips or number lines can help students see how different denominators indicate different fractional parts of the whole.



Slide 14: Mixing up addition and multiplication

Another mistake can occur when students take what they've learned about addition of fractions and apply it to multiplication.

Pictorial representations can help students see the meaning of two-thirds of one-third.



Slide 15: Mistakes with "invert and multiply"

Students quite often have difficulty with the "invert and multiply" procedure for dividing fractions. The problem arises when students are taught this procedure without an understanding of why the procedure works.

Students are less likely to make errors if the steps involved are clearly explained. They need to see that multiplying a number by its reciprocal yields a product of one. Then they need to understand

that dividing a number by one doesn't change the number. Once this is clear, teachers can show that "invert and multiply" involves multiplying both fractions by the reciprocal of the divisor.

Slide 16: Conclusion

Teachers should feel comfortable taking the time needed to build students' conceptual understanding as they also teach the procedures for performing operations with fractions.

Representations, estimations, and real-world contexts will help to strengthen students' grasp of fractional concepts.

Most important, teachers should expect students to make mistakes in computations with fractions. These mistakes can offer a window into students' thinking and an opportunity to use mistakes in learning.

Slide 17: Learn more

To learn more about Making Sense of Computational Procedures, please explore the additional resources on the Doing What Works website.