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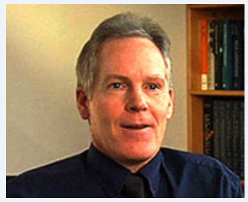
VIDEO
6:42 min

Full Details and Transcript



The Concepts Behind Operations

David C. Geary, Ph.D.

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Topic DEVELOPING EFFECTIVE FRACTIONS INSTRUCTION FOR K-8

Practice OPERATIONS WITH FRACTIONS

Highlights

» Dr. David Geary discusses why students have difficulty with operations with fractions, including not understanding that the procedures differ from those with whole numbers.

» He stresses the importance of students understanding the concepts underlying procedures, which enables them to recognize errors, estimate reasonable answers, and transfer knowledge to new problems,

» He provides examples of how the ability to estimate answers with fraction multiplication and addition would help children recognize their mistakes.

» Common misconceptions with fractions operations involve children applying what they already know to what they are trying to learn.

» Dr. Geary provides a real-world example of using coins/cents to illustrate the need for a common denominator when adding fractions. He also illustrates multiplication of fractions using a number line.

» He shows division of fractions in two ways, multiplying by reciprocals to show mathematically why the procedure works and using a number line to illustrate the procedure.

» Dr. Geary concludes by urging teachers to use students' mistakes to understand the misconceptions they harbor about fraction operations.

About the Interviewee

Dr. David C. Geary received a B.S. in psychology from Santa Clara University, an M.S. in child clinical/school psychology from California State University, and an M.A. and Ph.D. in developmental psychology from the University of California, Riverside. After completing his Ph.D. in 1986, he held faculty positions at the University of Texas at El Paso and the University of Missouri, first at the Rolla campus and then in Columbia. Dr. Geary served as chair of his department from 2002 to 2005 and as the University of Missouri's Middlebush Professor of Psychological Sciences from 2000 to 2003. He is currently a Curators' and Thomas Jefferson Professor. He has published nearly 200 articles, commentaries, and chapters across a wide range of topics, including three sole-authored books: *Children's Mathematical Development*; *Male, Female: The Evolution of Human Sex Differences* (now in second edition, 2010); and *The Origin of Mind: Evolution of Brain, Cognition, and General Intelligence*. He also co-authored *Sex Differences: Summarizing More Than a Century of Scientific Research*. He served as a member of the President's National Mathematics Advisory Panel and chaired the Learning Processes subcommittee, is a recipient of a MERIT award from the National Institutes of Health, and was appointed by President George W. Bush to the National Board of Directors for the Institute for Education Sciences. He currently directs the Missouri Longitudinal Study of Mathematical Development and Disability.

Full Transcript

00:05 I am Dave Geary, psychology professor at the University of Missouri. I contributed to the development of the fractions Practice Guide and was a member of the President's National Mathematics Advisory Panel.

The basic operations for fractions—that is, addition, subtraction, multiplication, and division—are the same as with whole numbers, but the procedures to get those results differ, and that is what can be confusing for kids.

00:34 It's important to understand the concepts underlying procedures in addition to correctly using the procedures. Understanding the concepts allows kids to recognize errors when they make errors. It allows them to estimate what is a reasonable answer and what's an unreasonable answer. And it's also important to for them to transfer their knowledge to new problems, that is, use fractions in areas where it's appropriate, but that they haven't actually had before.

01:11 If you understand that fractions are numbers and that seven-eighths is very close to one and that seven-sixths is a little over one, then if you are adding seven-eighths and seven-sixths, your answer should be very close to two. And so, if you get an answer very different from that, then you should recognize that you have done the adding incorrectly.

01:43 The same is true for multiplication—that if you are multiplying two fractions, say, one-half times one-third, you should know that the outcome is going to be less than both of those. It will be one-half of one-third.

Conceptually understanding what multiplication does and that fractions really are numbers, just like whole numbers, allows kids to recognize when they make these mistakes.

02:09 Misconceptions in fractions is very, very common. Many of those misconceptions result from kids trying to apply what they already know to what they are trying to learn. For fractions it often leads to errors. Children don't really understand the need for a common denominator, but they do understand related types of real-world examples. They understand that you can't add one quarter and one dime and get two quarters, or add one quarter and one dime and get two dimes, because they know that one quarter is 25, or one-fourth of a dollar, and one dime is ten, or one-tenth of a dollar. And they know that adding them up will give you 35.

02:58 I prefer using the number line over other types of pictorial or concrete examples. The number line is easily used to illustrate the need for a common denominator. So, for example, three-fourths plus one-half. Kids will often add the numerators and add the denominators—incorrectly coming up with four-sixths or two-thirds. If you take the number line and convert the one-half to two-fourths, then you can easily take that segment of two-fourths, add it to the other number line with the segment of three-fourths, and come up with the correct answer of five-fourths.

03:42 At times, it's easier to use different types of representations other than the number line. A good example of this is multiplication of fractions, particularly with values less than one. The fraction [Practice] Guide presents an example of a cake where the frosting available only covers two-thirds of the cake, and then you want to cut that frosted part into quarters, or one-fourths. The result then being you're getting not one-fourth of the cake, but one-fourth of two-thirds of the cake. Multiplying that through, you get two-twelfths, or one-sixth of the total cake, or one-fourth of the two-thirds of the cake. It's a good way of illustrating how multiplying two numbers less than one gives you an even smaller number than the two you started with.

04:38 When learning to divide fractions, children are taught the invert-and-multiply rule, which works, but they don't understand why it works. They don't understand the concept underlying it. We can approach teaching this two ways. One is a few steps to prove mathematically why it works, and the other is to use a number line to illustrate why it works.

05:03 The first step would be to multiply the denominator, one-fourth, by it's reciprocal. So one-half times four over one divided by one-fourth times four over one, and that gives us four-halves divided by one. Any number divided by one is itself, and that gives us four-halves, which equals two.

05:33 Now, if we took a number line and had the same problem, one-half plus one-fourth, and we broke the one-half into fourths, we could then ask the question: How many one-fourths fit into one-half? Then you could demonstrate on the number line that that would be two.

05:57 For kids to solve fractions problems correctly, they have to know the procedures and they have to know the concepts underlying those procedures. If they don't understand the concepts, they are not going to be able to use the procedures in different contexts.

06:13 Kids will make mistakes, and their mistakes will reflect their misconceptions. Use these errors and misconceptions to teach the concepts. It is a perfect way to understand why they are making mistakes and a perfect avenue for correcting those mistakes.