DOINGWHATWORKS

VIDEO
7:11 min

Full Details and Transcript

Learning to Think Proportionally

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Topic DEVELOPING EFFECTIVE FRACTIONS INSTRUCTION FOR K-8

Practice RATIO, RATE, PROPORTION

Highlights

- Dr. Lewis describes multiplicative relationships as the definition of thinking proportionally, illustrated by the example of enlarging a photo.
- Teachers can craft a journey of understanding for students, from proportional relationships with simple numbers to more complex numbers.
- Strategies for teaching proportion include build up, ratio table, and unit ratios.
- Ratio tables allow students to see different values that represent the same ratio and see that corresponding fractions are equivalent.
- Teachers should emphasize units to match the context because students often set up the wrong proportion when the fractions they write don't have meaningful units.
- Understanding that getting the right proportion is the central concept to teach.
- Dr. Lewis explains that cross-multiplication is a technique for solving proportion problems as numbers get more difficult.

He demonstrates why cross-multiplication works: It is about answering the question of when two fractions are equal or when one fraction is bigger.

» See additional examples of proportion problems.

About the Interviewee

W. James (Jim) Lewis, Ph.D., is the Aaron Douglas Professor of Mathematics at the University of Nebraska–Lincoln (UNL) as well as director of the school's Center for Science, Mathematics & Computer Education. Dr. Lewis is principal investigator for two National Science Foundation Math Science Partnerships, NebraskaMATH and the Math in the Middle Institute Partnership. He was chair of the Conference Board of the Mathematical Sciences committee that produced *The Mathematical Education of Teachers* and co-chair of the National Research Council committee that produced the report *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium*. Dr. Lewis was also co-principal investigator for Math Matters, a National Science Foundation grant to revise the mathematics education of future elementary school teachers at UNL.

Full Transcript

00:00 My name is Jim Lewis. I am professor of mathematics here at the University of Nebraska–Lincoln and also the director of the Center for Science, Math & Computer Education. Over the past year, year and a half, I have worked with the fractions panel to create a fractions practice guide for Doing What Works. When we encounter a situation we want to understand or a problem we want to solve, sometimes multiplicative relationships matter, and that's what people mean when they talk about thinking proportionally. You have a photograph, you want the photograph blown up to be on a poster that's going to advertise something. You know the photograph is about three inches tall, and when you blow it up it's going to be about 12 inches tall. It's shocking how many people don't realize the width changes as well.

00:59 We need to think about multiplicative relationships and the ratio between quantities when we are changing like this. In order for students to be successful and working problems that call for proportional reasoning, we have got to take them on a journey from where they are when they start to where we want them to get. To go on a journey so they understand the concepts involved, we want them to see this proportional relationship with much simpler numbers. We want them to see the build-up strategy and a ratio table. But then we want them to see their first problem that is what I called "going by way of one," or a unit ratio strategy. They see the context as the difficulty or the complexity of the numbers involved and the question being asked continues to increase.

01:58 And so they see the ratios that they want to set up with much simpler problems and much simpler questions. So as the problem becomes more complex, they use units of measure, the context, and past experience with similar problems to set up a ratio that is going to successfully lead them to the answer.

02:22 A teacher will craft the journey from a problem that's fairly easy to think about to one that's more involved. And then the teacher will begin to show the fractions that are in the problem, the two-fifths representing the two-to-five ratio. This ratio table is a technique that allows one to see several values that often represent the same ratio. And how corresponding fractions are equal can lay a foundation for students to understand context and to move in the direction of setting up ratios that become proportions. When we are saying a first ratio needs to equal a second ratio, then that's a proportion that we might want to solve.

03:15 Students know that they write fractions and compare fractions, but sometimes they don't quite know how they are supposed to compare them. Sometimes with very concrete number problems, we get into comparison and students don't understand what to do with the comparison. Students sometimes don't recognize the problem as the same mathematically if you change the context, or they don't

recognize it as the same mathematically if you made the numbers involved slightly more complicated than they are used to seeing.

03:53 If they have learned that they should set a proportion with the missing answer—the problem they want to solve as a variable—they sometimes set up a proportion, but they setup the wrong proportion because the fractions they write don't really mean anything to them.

04:12 It would help if teachers start emphasizing units. Sometimes I see students who choose the location simply wanting the variable to be in the numerator rather than matching up the context. If teachers emphasize the unit—flour to water or water to flour equals water to flour—then the student knows that they have to have the numbers and the variable match with those units.

04:42 This is typical of both the kind of problem students will run into—they know they want a proportion, but they don't know which proportion—and how teachers can work to minimize that kind of difficulty. The ratio table helps. Using units in the fractions that you've set up helps. As the problems get more difficult, we've got to set up a proportion and we've got to have a technique for solving that proportion, because we can't just think about the numbers in our head and reason our way to an answer.

05:20 But getting the right proportion is the central piece of the problem. What you do after that point is just technique. People refer to it as cross-multiplying. It's fairly straightforward, but if you're working with the wrong ratios, it's not going to give you the correct answer.

05:40 It's surprising how often I've worked with teachers and asked them, "Why does cross-multiplying work?" What I want them to understand is it really is about answering a question, "When are two fractions equal?" or "Which fraction is bigger?"

05:57 If I gave you the fractions 48 over 64 and 36 over 48, a sixth-grade student and perhaps many adults and some teachers do not quickly and easily answer that question, "What is the ratio, or what is

the fraction that they are looking at?" But they do learn a technique. If you have two fractions and if they have the same denominator, then they are equal only if they have the same numerator. So the goal is to move from the fractions you're working with to a pair of equivalent fractions that have the same denominator, and now you'll compare numerators. That's all cross-multiplication is. Those are the two numerators you get when you get a common denominator for the proportion that you set up.

06:46 As teachers are working with students, and they are beginning to work some problems correctly, the teacher wants to equip the student with enough strategies—essentially, enough tools in their toolbox—so that they will be able to solve a wide variety of problems.