DOINGWHATWORKS

Ratio, Rate, and Proportion Problems

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Topic: Developing Effective Fractions Instruction for K-8

Practice: Ratio, Rate, Proportion

Provided by Dr. James Lewis, this collection of 12 ratio, rate, and proportion problems includes comments on solution methods (e.g., buildup, unit rate ["going by way of 1"], or ratio table strategies) as well as cautions about potential difficulties.

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Ratio, Rate and Proportion Problems

1. When you buy orange juice in cans of frozen concentrate, the directions are to use one can of concentrate and three cans of water. Here are several questions one might ask:
   a. If you use 4 cans of concentrate, how many cans of water should you use?
   b. If you use 15 cans of water, how many cans of concentrate do you need to use?
   c. If you want to make 12 cans of orange juice, how many cans of concentrate and how many cans of water should you use?
   d. If you use 3 cans of concentrate and 8 cans of water, will the orange juice mixture be stronger or weaker than recommended?

A problem like this can often be solved just by reasoning about the multiplicative relation between the amount of concentrate and the amount of water. Here we would say that the ratio of concentrate to water is "one to three." Many people would use the notation 1:3 and others would say that the fraction 1/3 tells us the relation between concentrate and water.

We might use what is sometimes called a "buildup strategy". For example, if you make a second batch of orange juice, you will have used 2 cans of concentrate and 6 cans of water. Once a 3rd batch is made, you will have used 3 cans of concentrate and 9 cans of water. Students should be encouraged to use this line of reasoning and organize information into a Ratio Table:

Concentrate	1	2	3	4	5	6
Water	3	6	9	12	15	18
Orange Juice	4	8	12	16	20	24

We can read the answer to our questions off the Ratio Table:
   a. 12 cans of water
   b. 5 cans of concentrate
   c. 3 cans concentrate and 9 cans of water
   d. This time one might notice that the instructions would be to use 9 cans of water with 3 cans of concentrate. If we only use 8 cans of water, the orange juice should be slightly stronger than the recommended mixture. Note that mathematically this is the same as saying $\frac{1}{3} < \frac{3}{8}$.

This last answer might draw our attention to another approach to solving our problems. We say that the ratio, 1:3 is equivalent to a ratio like 6:18 because $\frac{1}{3} = \frac{6}{18}$. Similarly, we could answer question 1.a. by setting up the proportion $\frac{1}{3} = \frac{4}{w}$ and solving for a value for *w* that makes the statement true. Many students want to learn a technique, often referred to as "cross multiplication" to solve the problem, but in this particular situation, the fact

that $\dfrac{1}{3}$ is a 1:3 ratio would tell us that we want to multiply 4 by 3 to get an answer of 12, or the fact that the numerators involve a 1:4 ratio would tell us that the answer of 12 is found by multiplying 3 by 4.

Having started with a very straightforward problem, a teacher might slowly increase the complexity of the problem in one or more of the following ways:

- Change the values of the ratio, e.g., 2:7
- Ask questions not answered by an equivalent ratio found using a whole number multiple.
- Change the context of the problem.
- Change the values to use less familiar numbers.

Other strategies for solving ratio, rate, and proportion problems go by names such as:

- Double number lines
- Emphasizing units
- Unit ratio (going by way of 1)
- Set up a proportion
    - Comparing like quantities
    - Comparing unlike quantities
    - Using cross multiplication
- View the problem as a multiplication problem.

In a high school science class, students often work with problems that involve motion where the object in motion travels at a constant rate. For example, a car that travels at 60 miles/hour. Students are reminded that "distance equals rate times time" and they are taught to use the formula, $D = R \times T$.

The same relationship works whenever we have a rate. Thus, if we have a more general "amount/unit of quantity" we might say that our equation becomes:

Amount = Amount/unit of quantity × How many of the quantity

2. A race car that travels at a constant rate travels 15 miles in 6 minutes. If the car continues to travel at that rate,
    a. How far will it travel in 15 (25) minutes?
    b. How long does it take for the car to travel 20, 25, 50, 500 miles?

3. An artist creates a scale model for a planned sculpture. The scale model is 10 inches tall and has a 2 inch wide base. If the sculpture is to be 50 inches tall, what should be the width of the base?

4. Ms. Nice's 5th grade class wants to figure out how tall the school flagpole is. On a sunny day, the class goes outside and measures that the shadow of the flagpole is about 21 feet long. At the same time, Juan's shadow is 3 feet, 4 inches long. Juan is 4 feet, 3 inches tall. How tall is the flagpole?

5. Ollie measures the area of his kitchen floor and finds that it is 2400 square decimeters. He needs to know how many square meters it is. Please help him by answering the question, and explain your response so that it will make sense to him.

6. A painting that is 120 inches by 150 inches will be reproduced on a poster. Suppose that on the poster, the 120 inch side will become 20 inches long. Determine how long the 150 inch side will become on the poster.

7. A vertical pole that is 1 yard long casts a shadow that is 2 feet 6 inches long. At the same time, Heather's shadow is 3 feet 9 inches long. How tall is Heather?

8. A recipe for cookies calls for 3 cups of flour and 1¼ cups of water. If you want to use the recipe (i.e. the same ratio) but make cookies using 10 cups of flour, how much water should you use?

9. My recipe calls for 2 ½ cups of flour and makes enough pancakes for 6 people. I only have 1 cup of flour, but I have plenty of the other ingredients. What fraction of my recipe can I make? Will I have enough pancakes for 2 people?

10. Ashley wants to knit a scarf that is 5 ½ feet long. Each day she knits ¾ of a foot of the scarf. How many days will it take her to finish?

11. A recipe for cookies calls for 2 ½ cups of flour and ½ cup of sugar. If you adjust the recipe to use 4 cups of flour, how much sugar should you use?

12. In the small village of Williamstown, two-thirds of the men are married to women in the town. Also, three-fourths of the women are married to men in the town. What is the ratio of men to women in the village of Williamstown?

One caution I should offer is that this discussion has completely been about what should be called **direct** proportion. In general, one might say:

Two variables x, y are directly proportional (i.e., y is directly proportional to x) if there is a constant k such that $y = k \times x$. The value k is called the constant of proportionality. In our problems, it would be the rate.

There are other problems where two variables x, y are **inversely** proportional, i.e. if their product is a constant, k. In this case we would write: $x \times y = k$. An example of a problem that calls for this kind of relation is:

- Three people work at the same rate and paint a large room in 2 ½ hours. If only two of the people had shown up to work on the job, how long would we expect it to take them to paint the room?

  To solve this, we say that that

  3 people × 2 ½ hours $=$ 7 ½ people-hours $=$ 2 people × t hours

  The answer is t = 7 ½ ÷ 2 $=$ 3 ¾ hours.