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Understanding the Major Topics of School Algebra

Wilfried Schmid, Ph.D. • November 2008

Topic: National Math Panel: Major Topics of School Algebra

Highlights

- Overview of the major topics of school algebra
- Algebra as a foundation for higher mathematics and future success
- Why all students should learn algebra
- How the Panel identified the major topics of school algebra
- Relationships among algebra topics
- Example of why the topic of functions is important
- Is there a specific order of algebra topics that should be followed?

About the Interviewee

Wilfried Schmid is the Dwight Parker Robinson Professor of Mathematics at Harvard University. He was born in Germany and came to the US in 1960. He received his B.A. from Princeton University in 1964 and his Ph.D. in 1967 from the University of California, Berkeley. He taught at Berkeley and at Columbia University before joining the Harvard faculty in 1978. From 1990 until 1993 he served as chairman of the Harvard mathematics department.

Schmid's research revolves around Lie groups and their representations. He has introduced geometric methods in the study of infinite dimensional representations, and conversely has applied representation-theoretic methods in other areas of mathematics.

His intense interest in mathematics education arose in 1999, when confronted with a disturbing incident in his daughter's second grade math class. After publicizing the incident, he was invited to help revise the Mathematics Curriculum Framework for Massachusetts. Since then, Schmid has served as Mathematics Advisor to the Massachusetts Department of Education, as member of the Steering Committee of Mathematics NAEP, and as member of the Program Committee of the International Congress of Mathematics Education 2004. He has lectured widely on the subject of mathematics education, to audiences of mathematicians. Together with Deborah Ball, Jeremy Kilpatrick, Joan Ferrini-Mundi, Jim Milgram, and Richard Schaar, he wrote the declaration "Reaching for Common Ground in K-\12 Mathematics Education".

Full Transcript

My name is Wilfried Schmid. I am Professor of Mathematics at Harvard University. I was a member of the National Mathematics Advisory Panel. The panel did some of its work as a whole and some in five task groups and three committees. I was a member of two of the task groups: on critical knowledge and skills and the task group on assessment.

Algebra is really the gateway to mathematics, to engineering, to science in college. And not surprisingly, there are a number of studies that show a very high correlation between success in algebra in high school and graduation from four-year colleges. The subject matter of Algebra I is important for technical careers, even careers that don't require a college education. For students who expect to go to college to major in science, including biological science and social science, or in engineering or who expect to go on to a professional school, well they certainly also need Algebra II.

The panel was asked to make recommendations, based on the best available evidence, on the skills and skill progressions necessary for success in algebra. To do that, one, of course, has to have clarity on what algebra actually is. Three of the members of the panel, including myself, are university mathematicians, so we used, first of all, our professional judgment. We also examined the state standards of all 50 states—the technical term for these documents is "curriculum frameworks," and we looked at the curriculum guidelines of the high performing countries, such as Japan, South Korea, Singapore.

Now, there are some controversies in mathematics education, but the nature of algebra is not. There seems to be a high degree of consensus of the major topics of algebra. The panel gives a list of six major topics, which cover Algebra I, Algebra II, and depending on how the curriculum is organized also part of Pre-Calculus. The major topics are: symbols and expressions, linear equations, quadratic equations, functions, the algebra of polynomials, and combinatorics and finite probability. The first five topics really constitute the essence of algebra, and they are all equally important. The last topic, combinatorics and finite probability, is really an application of the binomial theorem. A very important application to be sure, but not really basic algebra. In all cases, it is not just the skills and concepts that should be taught, but also the way these are used in solving problems.

Algebra involves three main circles of ideas; symbolic computation, the notion of function, and the process of translating problems into equations that then can be solved. Our list of major topics of algebra is an elaboration of these circles of ideas. The idea that one can compute with symbols as if they were numbers—provided one uses the rules that apply to computations with numbers—is absolutely crucial to algebra and all the mathematics that comes after algebra.

The idea of function is related to symbolic computation. Let's imagine somebody standing on the top of a tall building and throwing a stone so it can fall freely. From basic physics or from observations that have been made, one knows that after one second, the stone has fallen approximately 16 feet; after two seconds, 64 feet; after three seconds, 144 feet. Now, one can make a list of how far the stone falls after various time intervals. But the far more efficient way is to say that the stone falls $16t^2$ feet when t is measured in seconds. And it's important to realize that t does not have to be an integer, it can be any real number. How long will it take the stone to hit the ground? Well, that depends of course on the height of the building. Let's say the height of the building is h measured in feet. Then, the amount of time, t, it takes to hit the ground satisfies the equation 16t2=h. And that can be solved for t. This is a very simple example, of course, but the process of translating the problem into an equation that can be solved is entirely typical for applications of algebra.

Is a traditional single subject approach more effective than an integrated approach? Well, in the abstract, this question is almost impossible to answer based on really solid evidence. High quality studies really do not look at the two approaches in general. They may compare individual curricula but not the approaches in general. What is important is that in either approach, the subject be presented coherently in a logical sequence and that enough attention is paid to the connections between various subjects. Symbols and expressions and linear equations are definitely Algebra I subjects. At the other end, complex numbers, the fundamental theorem of algebra, the binomial theorem and its applications are sometimes taught in Pre-Calculus. Everything in between belongs to Algebra II. For an integrated sequence, the grade level at which various topics are taught depends very much on how the curriculum is organized, how geometry topics and algebra topics are intertwined. But the sequence must be organized logically, and all of these topics are to be covered by the end of eleventh grade or the beginning of twelfth grade.

Serving on the National Mathematics Panel was a really interesting experience and I hope, as I am sure all of my colleagues on the panel do, that our recommendations can make a real difference to how Mathematics is taught in this country.