DOINGWHATWORKS

Presentation

FULL DETAILS AND TRANSCRIPT

Introducing Students to Logarithms

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Topic: National Math Panel: Major Topics of School Algebra

Practice: Topics of Algebra

Highlights

- Pitfalls that teachers might encounter when introducing logarithms
- Teaching pitfalls: prematurely focusing on history of logarithms, algebraic properties of logarithms
- Teaching pitfalls: working with an abstract base or a famous base of logarithms; approaching logarithms as inverse functions
- Introducing logarithms by building on what students know about graphing
- Setting up a table by choosing convenient values of x to graph a function
- Creating a graph of a function $y=\log_2 x$

About the Interviewee

Raegen T. Miller is a Senior Education Policy Analyst at American Progress. His work focuses on strategic management of human capital in education. He has published articles in peer-reviewed research journals shedding light on the productivity costs of teacher absences. Prior to joining American Progress, Raegen was a National Academy of Education/Spencer Postdoctoral Fellow affiliated with the Center on Reinventing Public Education at the University of Washington. He holds a doctorate

in Administration, Planning, and Social Policy from the Harvard Graduate School of Education, where he taught courses on applied data analysis and the foundations of schooling and teaching.

Raegen's work in education policy is grounded in many years of practice and service. He taught mathematics in the United States and abroad, in traditional public schools and in charter schools, and in urban and suburban settings. Raegen completed his teacher training at Stanford University, and he holds an M.S. in mathematics from Cal Poly, San Luis Obispo. He was a trustee of Prospect Hill Academy Charter School in Somerville, Massachusetts, and he served as president of his local teachers' union in Palo Alto, California.

Full Transcript

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Welcome to Introducing Students to Logarithms.

Slide #2

My name is Raegen Miller, and I am an education policy analyst. I taught math mostly at the high school level for about twelve years in both urban and suburban settings.

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I am offering a perspective as an experienced teacher, and mostly it's about the pitfalls that you can have when you are trying to teach students logarithmic functions. And we are talking about avoiding those dangers and instead leveraging successfully student knowledge and their prior experiences graphing functions.

Slide #4

The history of logarithms is mostly about logarithms as a tool for facilitating computations, and this is where slide rules and tables played a huge role in mathematics education, but is a completely obsolete point of view. And it's very easy for teachers to sort of go down this road, but it's quite a dangerous one because it gives students the wrong impression of what logarithmic functions are about because that aspect of logarithms is going to have nothing to do with their experience.

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The second danger is that it's very easy to dwell on the properties of logarithms, essentially algebraic properties, that are important but can be quite a side road that prevents students from getting acquainted with logarithms in a successful way initially. So for example, there is a famous conversion formula: $\log_c x = \log_b x / \log_b c$. So this is a way of converting logarithms from one base to another, and it's very important, but dwelling on this and doing exercises around this is not going to help students master the basic graph of logarithms. The third danger is that it's very tempting to work with an abstract base and do a lot of work initially with $\log_b x$ where *b* is some positive number. I recommend working with a concrete base first—something greater than one.

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The fourth danger is working with a famous base of logarithms. There are a couple of famous bases. Base 10 is very famous base for logarithms and base *e*, *e* is the base of the natural logarithm. The fifth danger is approaching logarithms as a matter of inverse functions, and inverse notation is sort of notoriously difficult; it's a little bit counterintuitive. Having the inverse notation in the same vicinity as a logarithmic expression when students are still not comfortable with a logarithmic expression and what it means at all is just a recipe for trouble.

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Logarithmic functions are just functions, and students have graphed many functions successfully, and we can take advantage of that fact to get them very quickly comfortable with the basic idea of a logarithmic functions graph. Let's use base two. So, this is $\log_2 x$, and that is a function we like to graph. And we will use that as the basis for getting kids comfortable with graphing, getting the essential graph of logarithmic functions. The special new thing is that this is a whole new notation. They need to learn to convert this expression, the logarithmic one, to an exponential form, which is equivalent. So, $y=\log_2 x$ is the same as $2^y=x$, and that latter expression is the one we can work with to set up a table of values that we can use to make a graph. And there is where we can leverage prior student knowledge because they have got a really good sense of habits that are involved with graphing a function.

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If they have the rule $y=1/3x+7$—this is something they have had clearly before they are going to ever wind up looking at logarithms—they know that if they want to make a table of values to start graphing this, the thing to do is choose values of *x* that are convenient because the way we are going to work with this new function that's been translated into an exponential expression does have a little bit of a twist to it. So here

again, I have $y=\log_2 x$, and we want to graph that, get the essential graph of that function. So the first step, remind them $2^y=x$ is the equivalent expression. Another way of putting it is, "What is the exponent that you would put on 2 in order to get *x*? So, *y* is the exponent we would put on 2 in order to get *x*?" and that's kind of a formulation to help remind them how to make that translation.

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Then we are going to set up a table, and as usual we want to choose values of *x* that are convenient. For students to think of the values of *x* that are convenient is little bit of a new experience. If *x* looks like 2 to some number, then it would be easy to pick out what *y* is. So, let's say *x* were 2^{-2}, that would be convenient because that means *y* would be -2 or if *x* were 2^{-1}, well you compare that with 2^y, so *y* is -1. What's different here, besides a little bit of a different clever step in picking what's convenient and thinking about what's convenient, is working out the numbers before we can go and actually graph them of the coordinates happens on different side. So on the table on the left side we have got to do well, "What is 2^{-2} and what is 2^{-1}?"

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Now, we have five nice clean points we can use to make a graph. I always label my axes, and here I have got the scales indicated adequately, and then graph the points. So, I have: 1,0; 1/2,-1; 1/4,-2; 2,1; 4,2, and those five points are sufficient here in these red dots to indicate where I should draw in a curve that passes through all those points. And I can imagine what happens as *x* gets larger, we come off to the right and the graph still keeps going up pretty gradually, and as *x* gets closer to the *y*-axis instead of being 1/4 it's even smaller, then the graph's going to be coming down and hugging the *y*-axis.

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Once the students have had some success graphing logarithmic functions, I would be interested in revisiting the specific famous bases, and also it makes the subject more alive to talk about the history of logarithms. So I would bring it into the equation but only after the students have had success. What I have done here is to focus on what the students know and how that can be useful in broaching a new very challenging topic.

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To learn more about Introducing Students to Logarithms, please explore the additional resources on the Doing What Works website.