

Topic: Developing Effective Fractions Instruction for K-8 (Fractions)

Topic inventories list every component of a DWW topic. Use this document to get an overview of the Developing Effective Fractions Instruction for K-8 topic, identify multimedia pieces, and plan for professional development.

Topic Overview and Tools

State and District Examples

Fraction Concepts

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Fractions as Numbers

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Fraction Operations

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Ratio, Rate, and Proportion

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Topic Overview and Tools

Resource	Description	Related Media and Files
Developing Effective Fractions Instruction for K-8 (Video, 6:29 min)	This overview explains the importance of rational numbers and introduces the recommendations on effective fractions instruction in elementary and middle school, including the needed support and training for teachers.	Fractions Instruction Visual Diagram (.pdf) Developing Effective Fractions Instruction for Kindergarten Through 8th Grade (Practice Guide) (.pdf)
Fractions Instruction Visual Diagram (.pdf)	This diagram provides an overview of the instructional practices for teaching fractions in elementary and middle school, along with the information on the knowledge teachers need to teach fractions effectively.	
The Importance of Fractions Instruction (Video, 5:36 min) Robert S. Siegler, Carnegie Mellon University	Robert Siegler discusses why competence with fractions is critical for more advanced math and why U.S. students have difficulty grasping fractions. He also describes the developmental sequence that grounds learning about fractions in a solid base of conceptual understanding.	Transcript and Bio (.pdf)
What Teachers Need to Know About Teaching Fractions (Video, 6:32 min) Francis (Skip) Fennell, McDaniel College	Skip Fennell describes the knowledge teachers need to teach fractions effectively, paying special attention to multiple strategies for representing problems involving fractions. He discusses the convergence of curriculum and explores why students and teachers have difficulty with certain fraction concepts.	Transcript and Bio (.pdf)
Comprehensive Planning Template for Schools (.doc)	Template for working with school administrators and math staff in the district to work through school-level options for improving instruction in rational numbers.	
Comprehensive Planning Template for Districts (.doc)	Template to help district leaders apply the IES Practice Guide recommendations to the design of professional development and practical policies that facilitate improvements in instruction.	
Comprehensive Planning Template for States (.doc)	Template of strategies for facilitating conversations with state education agencies about setting a policy climate for improving teacher quality in math, specifically related to rational numbers.	
Planning Fractions Instruction Using Common Core State Standards for Mathematics (.doc)	A tool to identify which DWW items best illustrate the content and skills of the individual Common Core mathematics standards that are relevant to the teaching of fractions.	



State and District Examples

Resource	Description	Related Media and Files
A Learning Trajectory for Fractions (Video, 5:22 min) Renee Sherry, Ken Jensen, & Kim Pippenger, Tollgate Elementary School, Aurora, CO	Mathematics coaches discuss developing children's understanding of fractions. An iceberg graphic organizer is used to unpack concepts students need to master. Early foundational concepts are identified.	Transcript (.pdf) Site Profile (.pdf)
Multiple Interpretations of Fractions (Video, 5:33 min) Jonathan Brendefur, Eliza Hart Spalding School of Math and Technology, Boise, ID	Dr. Brendefur describes professional development about fractions and describes a measurement approach to interpreting fractions using a number line.	Transcript (.pdf) Partitive and Measurement Fraction Problems (.pdf) Site Profile (.pdf)
Partitive and Measurement Fraction Problems (.pdf) Eliza Hart Spalding School of Math and Technology, Boise, ID	Two slides that describe the difference between partitive and measurement fraction problems and how to solve them.	Multiple Interpretations of Fractions (Video) Site Profile (.pdf)



Practice: Build basic fraction concepts from students' informal understanding of sharing and proportion. (Fraction Concepts)

Children have an intuitive understanding of basic fraction concepts, such as dividing a whole object into equal pieces or sharing a set of objects among a group of people, by as early as preschool. Teachers can structure activities that create connections between this informal knowledge of sharing and the formal fraction concepts of ordering and equivalence relationships. Informal sharing can be presented in terms of division (e.g., partitioning objects into groups of the same size) or ratio (e.g., the number of objects in relation to the number of people sharing).

Resource	Description	Related Media and Files
The Conceptual Basis for Fractions (Video, 4:46 min)	This multimedia overview introduces how children's intuitive understandings of sharing and proportion can serve as a foundation on which to build formal instruction of fraction concepts. This practice contains three tools, including a professional development sequence, planning tool, and lesson prompts.	Transcript (.pdf)
Learning Together About Building on Informal Understandings of Fractions (.doc)	Activities in this professional development tool will help primary grade teachers learn about working with young children on informal fraction concepts.	Creating Story Problems: Initial Fraction Concepts (.doc) Discussion Prompts: Initial Fraction Concepts (.doc) Developing Effective Fractions Instruction (Practice Guide) (.pdf) The Conceptual Basis for Fractions (Video) A Learning Trajectory for Fractions (Video) Equal Sharing in Grade 1 (Video)
Creating Story Problems: Initial Fraction Concepts (.doc)	Planning tool to create problems that build on children's informal understanding of fractions. Includes components for constructing a progression of problems.	The Conceptual Basis for Fractions (Video) A Learning Trajectory for Fractions (Video)



Resource	Description	Related Media and Files
Discussion Prompts: Initial Fraction Concepts (.doc)	Discussion prompts used to help children extend their thinking during formal encounters with fractions.	The Conceptual Basis for Fractions (Video)
		Building on Intuitive Understanding (Video)

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Resource	Description	Related Media and Files
Building on Intuitive Understanding (Video, 6:51 min)	Dr. Carpenter discusses building children's basic fraction understanding by building on their intuitive knowledge of sharing.	Transcript and Bio (.pdf)
Thomas P. Carpenter, University of Wisconsin-Madison	Children initially solve problems by representing the sharing action using drawings or counters. Sharing is used to develop the concept of equivalence of fractions.	

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Resource	Description	Related Media and Files
Representations of Part-Whole Relationships (Video, 6:44 min) Sorsha Mulroe, Howard County Public Schools, Ellicott City, MD	An elementary math coach explains fair sharing. Students use counters and pictorial representation to solve a fair sharing problem. Students use a fraction bar to represent their solution, post it on a board, and explain it to peers. Terms like whole, notation, numerator, and denominator are introduced to students.	Transcript (.pdf) Fractions With Cuisenaire Rods (.pdf) Site Profile (.pdf)
Fractions With Cuisenaire Rods (pdf) Howard County Public Schools, Ellicott City, MD	This worksheet is used by students when exploring equivalent fractions using Cuisenaire Rods. The teacher guides discussions using students' "How do you know?" responses.	Representations of Part-Whole Relationships (Video) Site Profile (.pdf)



Resource	Description	Related Media and Files
Teaching Fractions in Grade 2 (Video, 6:22 min) Kathy Lembo, Worthington Hooker School, New Haven, CT	A second-grade teacher describes a lesson used to review fractions, decimals, and percents in different waysthrough portions of area, clock, money, measurement, Unifix cubes—and assess student understanding of fractions.	Transcript (.pdf) Site Profile (.pdf)
Using Multiple Representations to Teach Fractions (slideshow) Mountain Ridge Middle School & Northridge Elementary School, Highlands Ranch, CO	Elementary and middle school teachers illustrate different ways to use multiple representations of fractions. Manipulatives, visual representations, and technology are used to develop students' conceptual understanding and fluency with fractions. Familiar materials, like egg cartons, can be effectively used across grades, moving from understanding parts of the whole to mixed fractions and computation.	Transcript (.pdf)
Equal Sharing in Grade 1 (Video, 6:46 min) Juliet Franklin, Tollgate Elementary School, Aurora, CO	A first-grade teacher guides students through a problem set requiring fair sharing of objects and sets of objects. She explains building on the language of fractions.	Transcript (.pdf) Sharing Cookies Worksheets (.pdf) Site Profile (.pdf)
Sharing Cookies Worksheets (.pdf) Tollgate Elementary School, Aurora, CO	These worksheets focus on creating equal shares. They are differentiated to include problems of three different difficulty levels.	Equal Sharing in Grade 1 (Video, 6:46 min) Site Profile (.pdf)
A Learning Trajectory for Fractions (Video, 5:22 min) Renee Sherry, Ken Jensen, & Kim Pippenger, Tollgate Elementary School, Aurora, CO	Mathematics coaches discuss developing children's understanding of fractions. An iceberg graphic organizer is used to unpack concepts students need to master. Early foundational concepts are identified.	Transcript (.pdf) Site Profile (.pdf)
Same Parts, Different Whole: Lesson Plan and Worksheets (.pdf) Tollgate Elementary School, Aurora, CO	A fourth-grade teacher's lesson on finding fractional parts and comparing fractional parts of different-sized wholes. Questions to support struggling students are included.	Site Profile (.pdf)



Practice: Use number lines to develop student understanding of fractions as numbers. (Fractions as Numbers)

Understanding that fractions are numbers with magnitudes that can be ordered or considered equivalent is fundamental to grasping operations with fractions. Teachers can develop students' understanding of fractions as numbers by using number lines to illustrate magnitudes, the relation between whole numbers and fractions, and the relations among fractions, decimals, and percents.

Resource	Description	Related Media and Files
Recognizing Fractions as Numbers (Video, 5:40 min)	This overview explains why number lines are a central representational tool to teach fractions. Students need to learn that fractions are numbers with magnitudes that represent quantities. It is a conceptual basis for all operations with fractions.	Transcript (.pdf)
Learning Together About Number Lines (.doc)	Activities in this professional development tool will help primary- and intermediary-grade teachers understand the versatility of number lines in explaining fraction concepts.	Number Lines: A Key Representational Tool (Video) The Concepts Behind Operations (Video) Fraction Game on Number Lines (Slideshow) Multiple Interpretations of Fractions (Video) Fractions on a Number Line (Video)
Teacher-Coach Reflection on Practice: Fractions as Numbers (.doc)	Self-assessment tool for teachers to reflect upon strategies for improving students' understanding of fractions as numbers with magnitudes that extend whole numbers.	Developing Effective Fractions Instruction (Practice Guide) (.pdf) Number Lines: A Key Representational Tool (Video) A Teacher's Perspective: How to Use the Practice Guide (Audio)



Resource	Description	Related Media and Files
Observing Students' Understanding Through Fraction Games (.doc)	An observation tool for monitoring students' understanding of fraction concepts while playing fraction games.	Fraction Strip Game: Cover Up (.pdf)
		Fraction Tracks Game (.pdf)
		Making and Using Fraction Strips (Video)
		Fractions on a Number Line (Video)

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Resource	Description	Related Media and Files
Number Lines: A Key Representational Tool (Video, 6:45 min)	Okamoto describes the misconceptions that students have about fractions and how to help them understand fractions' place in the number system.	Transcript and Bio (.pdf)
Yukari Okamoto, University of California, Santa Barbara	She recommends number lines as a robust representational model and useful for adding fractions and demonstrates measurement activities with paper strips and parallel number lines portioned into fractional parts.	
A Teacher's Perspective: How to Use the Practice Guide (Audio, 4:37 min) Laurie Thompson, IES Panel Member	Thompson suggests for teachers to review the fraction practice in the IES guide associated with their grade level. The guide emphasizes the use of number lines and highlights visual representations, examples, roadblocks, and practical solutions.	Transcript and Bio (.pdf)

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Resource	Description	Related Media and Files
Fraction Game on Number Lines (Slideshow) Tollgate Elementary School, Aurora, CO	Fifth graders play the Fraction Tracks game to practice equivalent fractions. Students build the game board, which helps with placing equivalent fractions on parallel number lines. Variations on the game parallel students' fluency with fractions.	Transcript (.pdf) Fraction Tracks Game (.pdf) Site Profile (.pdf)



Resource	Description	Related Media and Files
Fraction Tracks Game (.pdf) Tollgate Elementary School, Aurora, CO	This game helps students understand equivalent fractions, and mentally add and subtract fractions. Included are the game boards, rules for basic game play, and "extension" rules that provide more advanced practice.	Fraction Game on Number Lines (Slideshow) Site Profile (.pdf)
Using a Number Line to Teach Fractions (Video, 6:00 min) Christian Skalstad, Madison Elementary School, Spokane, WA	A third-grade teacher and an instructional math coach demonstrate the use of an open number line (ONL) for moving beyond counting. Students can use an ONL as a tool in manipulating whole numbers and fractions.	Transcript (.pdf) Frank's Fresh Farm Produce (.pdf) Site Profile (.pdf)
Frank's Fresh Farm Produce (.pdf) Madison Elementary School, Spokane, WA	A word problem student groups can complete using a double number line. The sample includes photos of posters students developed to demonstrate their solutions.	Using a Number Line to Teach Fractions (Video) Site Profile (.pdf)
Making and Using Fraction Strips (Video, 5:47 min) Cara Crist, Eliza Hart Spalding School of Math and Technology, Boise, ID	A third-grade teacher uses fraction strips to emphasize fractions. A game called Cover Up is used to practice finding equivalent fractions.	Transcript (.pdf) Fraction Strip Game: Cover Up (.pdf) Site Profile (.pdf)
Fraction Strip Game: Cover Up (.pdf) Eliza Hart Spalding School of Math and Technology, Boise, ID	This game helps students understand unit fractions and equivalents. Students create the game board and pieces required to play.	Making and Using Fraction Strips (Video) Site Profile (.pdf)
Fractions on a Number Line (Video, 4:32 min) Vonda Franklin, Eliza Hart Spalding School of Math and Technology, Boise, ID	A fourth-grade teacher presents a lesson to have students understand that fractions are numbers. Measurement activities are used to address misconceptions about fractions.	Transcript (.pdf) Site Profile (.pdf)



Resource	Description	Related Media and Files
Multiple Interpretations of Fractions (Video, 5:33 min) Jonathan Brendefur, Eliza Hart Spalding School of Math and Technology, Boise, ID	Professor Jonathan Brendefur describes how he helps teachers understand and teach different interpretations of fractions. He explains the importance of explicitly teaching about various interpretations and discusses how number lines can be used at each stage.	Transcript (.pdf) Partitive and Measurement Fraction Problems (.pdf) Site Profile (.pdf)
Partitive and Measurement Fraction Problems (.pdf) Eliza Hart Spalding School of Math and Technology, Boise, ID	Two slides that describe the difference between partitive and measurement fraction problems and how to solve them.	Multiple Interpretations of Fractions (Video) Site Profile (.pdf)



Practice: Help students understand why computational procedures with fractions make sense. (Fraction Operations)

In order for students to become proficient with computational procedures when working with fractions, they need a strong understanding of why those procedures make sense. Teachers should focus on building conceptual understanding and developing procedural fluency, emphasizing that the two are connected. This teaching for understanding requires teachers to have deep knowledge of computational procedures, be fluent with a range of representations, and know common misconceptions that interfere with students' understanding operations with fractions.

Resource	Description	Related Media and Files
Making Sense of Computational Procedures (Video, 6:09 min)	This overview explores the importance of focusing on conceptual understanding and procedural fluency with fraction operations and how they connect. It describes recommended instructional practices for developing understanding of computational procedures and ways to address typical student misconceptions.	Transcript (.pdf)
Learning Together About Students' Misconceptions About Fraction Operations (.doc)	A tool for an in-service on creating awareness about students' potential misconceptions about fraction operations.	Making Sense of Computational Procedures (Video) The Concepts Behind Operations (Video) Multiply or Divide? (Video)
Walkthroughs: Teaching Fraction Operations (.doc)	An observational walkthrough tool used in elementary and middle school math classes.	Developing Effective Fractions Instruction (Practice Guide) (.pdf) Supporting Teachers' Conceptual Understanding (Audio)
Planner: Meaningful Student Assignments With Fraction Operations (.doc)	A planner to help teachers develop practice problems in adding and multiplying fractions based in real-world situations.	Developing Effective Fractions Instruction (Practice Guide) (.pdf) Making Sense of Computational Procedures (Video) Multiply or Divide? (Video)



Learn What Works

Resource	Description	Related Media and Files
The Concepts Behind Operations (Video, 6:42 min) David C. Geary, Ph.D., University of Missouri	Dr. Geary describes building students' understanding of concepts underlying operations with fractions. He demonstrates teaching fractions using a real-world example and a number line.	Transcript and Bio (,pdf)
Supporting Teachers' Conceptual Understanding (Audio, 4:54 min)	A district instructional facilitator describes walkthroughs used as professional development.	Transcript and Bio (.pdf)
Jonathan Wray, Howard County Public School System	Coaches demonstrate lessons, co-teach, lead professional development, and assist with using data to inform practice.	
	Small professional learning communities empower teachers to examine their practices.	

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Resource	Description	Related Media and Files
Grade 5 Number Talk (Video, 6:16 min) Cindy Matthews, Tollgate Elementary School, Aurora, CO	A fifth-grade teacher teaches a lesson on adding and subtracting fractions and models number talk techniques. Number talk gives students an authentic situation to think about fractions.	Transcript (.pdf) Dumb Friends League Problem (.pdf) Site Profile (.pdf)
Dumb Friends League Problem (.pdf) Tollgate Elementary School, Aurora, CO	A math problem, which is centered on finding shelters for dogs, requires students to combine fractions and compare the sum.	Grade 5 Number Talk (Video) Site Profile (.pdf)
Ways to Measure 1 ½ Cups (Video, 6:02 min) Keith Phelps, Eliza Hart Spalding School of Math and Technology, Boise, ID	A fourth-grade teacher teaches a lesson on decomposing fractions. Students use blocks, draw pictures, and write a number sentence to identify different ways to make one and a half cups using different-sized measuring cups.	Transcript (.pdf) Site Profile (.pdf)



Resource	Description	Related Media and Files
Solving a Real-World Fraction Division Problem (Video, 6:41 min) Brett Mosely, Eliza Hart Spalding	A fifth-grade teacher uses a division problem to challenge students to find different approaches to a solution. Students use a variety of tools to find a solution.	Transcript (.pdf) Web Shots for Spiderman Problem (.pdf)
School of Math and Technology, Boise, ID	Students talk through their approaches with peers.	Site Profile (.pdf)
Web Shots for Spiderman Problem (.pdf) Eliza Hart Spalding School of Math and Technology, Boise, ID	A problem used in a lesson on division with fractions. Also includes examples of students' work.	Solving a Real-World Fraction Division Problem (Video) Site Profile (.pdf)
Multiply or Divide? (Video, 5:50 min) Clare Heidema, RMC Denver Professional Development, Aurora, CO	A facilitator works with elementary math coaches on fraction multiplication and division concepts. The partitive notion and measurement notion of division are described. Teachers write a real-world story problem for the partitive example.	Transcript (.pdf) Focus on Fraction Operations (.pdf) Multiply or Divide Problems (.pdf)
Focus on Fraction Operations (.pdf) RMC Denver Professional Development, Aurora, CO	A PowerPoint for a staff development session on how to teach fraction operations.	Multiply or Divide? (Video)
Multiply or Divide Problems (.pdf) RMC Denver Professional Development, Aurora, CO	A problem set of fraction multiplication and division concepts used in a professional development session.	Multiply or Divide? (Video)



Practice: Develop understanding of proportional relationships before teaching computational procedures. (Ratio, Rate, Proportion)

Proportional thinking—understanding multiplicative relationships between quantities—is essential for more advanced work in mathematics. Teachers should develop students' understanding of proportional reasoning before teaching the cross-multiplication algorithm as a procedure for solving proportions. Teachers can make connections among problem contexts involving ratios, rates, and proportions, and discuss which ones can be solved most easily with cross-multiplication.

Resource	Description	Related Media and Files
Developing Proportional Reasoning (Video, 7:16 min)	This multimedia overview outlines students' development of proportional thinking and its relationship to cross-multiplication.	Transcript (.pdf)
	A buildup strategy can be used to show equivalent ratios by the repeated addition or partitioning of the numbers in a ratio.	
	Students can use a ratio table to organize their thinking and record the relations in proportion problem.	
Learning Together About Teaching Ratios and Proportional Reasoning (.doc)	An in-service tool for giving teachers information on instructional strategies related to ratios, rates, and proportions.	Focus on Ratios (.pdf)
		Ratio, Rate, and Proportion Problems (.pdf)
		Ratio Problems (.pdf)
		Build-Up Strategies Worksheet (.pdf)
		Developing Proportional Reasoning (Video)
		Learning to Think Proportionally (Video)
		Ratio, Rate, Proportion Problems (Video)
		Understanding Ratios (Video)



Resource	Description	Related Media and Files
Analyzing Student Work: Ratios (.doc)	A tool for math coaches and teachers for reviewing student work with ratios and identifying areas they have grasped and which areas they may be struggling in.	Ratio: Warm Up Activity (,pdf) Build-Up Strategies Worksheet (,pdf) Ratio Problems (,pdf) Developing Proportional Reasoning (Video) Understanding Ratios (Video) Cross-Multiply? Not So Fast (Video)
Planning Real-World Problems for Ratio, Rate, Proportion (.doc)	A planning tool for middle-grade teachers to develop real-world context practice problems using ratios, rates, and proportions.	Developing Effective Fractions Instruction (Practice Guide) (.pdf) Learning to Think Proportionally (Video) Ratio, Rate, Proportion Problems (Video)

Learn What Works

Resource	Description	Related Media and Files
Learning to Think Proportionally (Video, 7:11 min) W. James (Jim) Lewis, University of Nebraska-Lincoln	Dr. Lewis discusses thinking proportionally and offers examples in everyday contexts. Strategies for teaching proportion include build up, ratio table, and unit ratios. Cross-multiplication is used for solving proportion problems as numbers get more difficult. A demonstration of why cross-multiplication is presented.	Transcript and Bio (.pdf)
Ratio, Rate, Proportion Problems (Video, 4:40 min) W. James (Jim) Lewis, University of Nebraska-Lincoln	Dr. Lewis demonstrates working through two examples of problems using ratios. It is important to show a problem worked out in more than one way.	Transcript and Bio (.pdf) Ratio, Rate, and Proportion Problems (.pdf)



Resource	Description	Related Media and Files
Cross-Multiply? Not So Fast (Video, 6:11 min) James Ro & Jackie Price, Howard County Public Schools, Ellicott City, MD	Eighth-grade co-teachers describe team teaching in an inclusion math classroom. Students use concrete materials, build-up strategies, and cross-multiplication to solve real-world problems. Students work in pairs to teach solutions and explain their thinking aloud.	Transcript (.pdf) Ratio: Warm Up Activity (.pdf) Build-Up Strategies Worksheet (.pdf) Build a Cafe: PowerPoint and Report (.pdf) Site Profile (.pdf)
Ratio: Warm Up Activity (.pdf) Howard County Public Schools, Ellicott City, MD	A three-problem activity middle school students complete using blocks as visual representations.	Cross-Multiply? Not So Fast (Video) Site Profile (.pdf)
Build-Up Strategies Worksheet (.pdf) Howard County Public Schools, Ellicott City, MD	A worksheet for middle school students on using build-up strategies when working on ratio problems.	Cross-Multiply? Not So Fast (Video) Site Profile (.pdf)
Build a Cafe: PowerPoint and Report (.pdf) Howard County Public Schools, Ellicott City, MD	A series of real-world problems students need to solve using various types of math, including ratios. This unit requires students to solve problems related to building a café.	Cross-Multiply? Not So Fast (Video) Site Profile (.pdf)
Understanding Ratios (Video, 8:00 min) Arlene Mitchell, RMC Denver Professional Development, Aurora, CO	A staff developer gives a workshop on the meaning of ratios and how they are linked to fractions. Students should be presented with different contexts to build up to working with proportions. Participants explain their reasoning. Mitchell also scaffolds their thinking and provides alternative ways to approach the problem.	Transcript (.pdf) Focus on Ratios (.pdf) Ratio Problems (.pdf)



Resource	Description	Related Media and Files
Focus on Ratios (,pdf) RMC Denver Professional Development, Aurora, CO	A PowerPoint from a professional development session focused on issues related to working with ratios.	Understanding Ratios (Video)
Ratio Problems (.pdf) RMC Denver Professional Development, Aurora, CO	Two word problems teachers in a professional development session complete to strengthen conceptual understanding of fraction operations.	Understanding Ratios (Video)
Ratio, Rate, and Proportion Problems (.pdf) University of Nebraska-Lincoln	A collection of twelve ratio, rate, and proportion problems along with solution methods and comments on potential difficulties students may have.	Ratio, Rate, Proportion Problems (Video)

