DOINGWHATWORKS

SAMPLE MATERIAL

K-8 Essential Mathematics Concepts and Skills

Cornell Elementary School, Iowa

Topic: Response to Intervention in Elementary-Middle Math

Practice: Foundations of Arithmetic

Teachers at Cornell Elementary School use the Iowa Core Curriculum to identify and direct their use of effective instruction and assessment of essential concepts in literacy, mathematics, science, social studies, and 21st century skills for K-12 students. The core curriculum sets high expectations for all students and respects that students will progress at different rates.

The first section of the *K-8 Essential Mathematics Concepts and Skills*[1] shows the essential math concepts and skills, content standards, and performance standards for grade spans K-2, 3-5, and 6-8. The second section provides details and examples for the essential concepts and skills for these grade spans.

1 Excerpted from the *Iowa core curriculum: K-12 mathematics*, Iowa Department of Education, http://www.corecurriculum.iowa.gov/

This project has been funded at least in part with Federal funds from the U.S. Department of Education under contract number ED-PEP-11-C-0068. The content of this publication does not necessarily reflect the views or policies of the U.S. Department of Education nor does mention of trade names, commercial products, or organizations imply endorsement by the U.S. Government.

Iowa Core Curriculum

The Iowa Core Curriculum provides a guide to delivering challenging and meaningful content to students that prepares them for success in life. The Iowa Core Curriculum identifies essential concepts and skills for kindergarten through 12th grade in literacy, mathematics, science, social studies, and 21st century skills. It also includes direction for teachers regarding effective instruction and assessment.

Grade Spans in the Iowa Core Curriculum

The Iowa Core Curriculum presents the most important concepts and skills that students should learn in grade spans, such as K-2, 3-5, and 6-8. This was done to honor the individual needs of students. The Iowa Core Curriculum sets high expectations for all students and respects that students will progress differently through the grade spans.

Mathematics Core Curriculum: Grades K-2	
Essential Concepts and/or Skills for Grades K-2	
Number and Operations	Count, represent, read, compare, order and conserve (knows that the total number does not change when configured differently) whole numbers. Develop understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts. Express numbers as equivalent representations to fluently compose and decompose numbers (putting together and taking apart). Develop fluency and quick recall of addition facts and related subtraction facts and fluency with multi-digit addition and subtraction. Estimate the answer to an addition or subtraction problem before computing, and determine whether the computed answer makes sense. Develop an understanding of whole number relationships, including grouping in tens and ones and apply place-value concepts. Understand fractional parts are equal shares or equal portions of a whole unit (a unit can be an object or a collection of things).

* Excerpted from the Iowa Core Curriculum: K-12 Mathematics, Iowa Department of Education, 400 E 14th St, Des Moines, IA 50319-0146, http://www.corecurriculum.iowa.gov/

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Mathematics Core Curriculum: Grades 3-5	
Essential Concepts and/or Skills for Grades 3-5	
Number and Operations	Develop an understanding of multiplication and division concepts and strategies for basic multiplication facts and related division facts. Develop fluency and quick recall of multiplication facts and related division facts and fluency with multi-digit multiplication and division. Develop the ability to estimate the results of computation with whole numbers, fractions or decimals and be able to judge reasonableness. Extend place value concepts to represent and compare both whole numbers and decimals. Use benchmarks to help develop number sense. Develop an understanding of commonly used fractions, decimals, and percents, including recognizing and generating equivalent representations. Develop an understanding of and fluency with addition and subtraction of fractions and decimals.
Content Standards for Grades 3-5	
Content Standard 1: Students can understand and apply a variety of math concepts.	
Benchmark: Students can understand and apply number properties and operations.	
Grade Level Indicator : Represent, compare, and order numbers	
Grade Level Indicator : Describe and apply properties of numbers	
Grade Level Indicator : Classify numbers by divisibility	
Grade Level Indicator : Demonstrate ways of performing operations	
Grade Level Indicator : Use place value; write numbers in standard, expanded, and exponential form	
Grade Level Indicator : Use and interpret operational and relational symbols	
Benchmark: Students can understand and apply concepts and procedures of algebra.	
Grade Level Indicator : Solve equations and inequalities	
Grade Level Indicator : Use variable expressions to model situations	
Grade Level Indicator : Explore numerical patterns	
Benchmark: Students can understand and apply concepts of geometry.	
Grade Level Indicator : Identify, classify, and compare geometric figures	
Grade Level Indicator : Describe geometric properties, patterns, and relationships	
Grade Level Indicator : Apply the concepts of perimeter, area, and volume	
Benchmark: Students can understand and apply concepts of measurement.	
Grade Level Indicator : Measure length/distance, time, temperature, weight, mass, and volume	
Grade Level Indicator : Estimate measurements with appropriate precision	
Grade Level Indicator : Identify and use appropriate units of measurement	
Benchmark: Students can understand and apply concepts in probability and statistics.	
Grade Level Indicator : Apply probability concepts and counting rules	
Grade Level Indicator : Understand and apply measures of central tendency and variability	

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Content Standard 2: Students can understand and apply methods of estimation.
Benchmark: Students can understand and apply concepts and procedures of standard rounding, order of magnitude, and number sense.
Grade Level Indicator : Use standard rounding to estimate
Grade Level Indicator : Use order of magnitude to estimate
Grade Level Indicator : Use number sense to estimate
Content Standard 3: Students can solve a variety of math problems.
Benchmark: Students can solve math problems.
Grade Level Indicator : Solve single-step and multiple-step math problems
Grade Level Indicator : Identify extraneous or insufficient information in problems
Benchmark: Students can understand and apply problem-solving approaches and procedures.
Grade Level Indicator : Choose a method for solving a problem
Content Standard 4: Students can interpret data presented in a variety of ways.
Benchmark: Students can use tables and graphs to locate and read information.
Grade Level Indicator : Locate amounts in specific cells of a table
Benchmark: Students can interpret data from a variety of sources.
Grade Level Indicator : Read amounts on scales of bar and line graphs
Grade Level Indicator : Compare quantities to determine ranks, sums, or differences and to find ratios
Grade Level Indicator : Use tables and graphs to determine rates or identify trends, understand underlying or functional relationships, and generalize or draw conclusions
Performance Standards for Grades 3-5
High Performance Level: Understands math concepts, solves word problems, and interprets data from graphs and tables. Usually can use estimation methods. ▪ Distinguished: Understands math concepts, solves word problems, and interprets data from graphs and tables. Usually can use estimation methods. ▪ Accomplished: Understands math concepts, solves word problems, and interprets data from graphs and tables. Usually can use estimation methods. **Intermediate Performance Level:** Usually can understand math concepts, solve word problems, and interpret data from graphs and tables. Sometimes is able to use estimation methods. ▪ Skilled: Usually can understand math concepts, solve word problems, use estimation methods, and interpret data from graphs and tables. ▪ Moderate: Sometimes can understand math concepts and use estimation methods. Usually is able to solve word problems and interpret data from graphs and tables. **Low Performance Level:** Seldom can understand math concepts, solve word problems, or use estimation methods. Sometimes can interpret data from graphs or tables. ▪ Marginal: Sometimes can understand math concepts, use estimation methods, and interpret data from graphs and tables. Seldom is able to solve word problems. ▪ Weak: Seldom can understand math concepts or solve word problems. Rarely can use estimation methods or interpret data from graphs and tables.

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Mathematics Core Curriculum: Grades 6-8	
Essential Concepts and/or Skills for Grades 6-8	
Number and Operations	Understand, apply, and be computationally fluent with multiplication and division of fractions and decimals. Understand, apply, and be computationally fluent with rational numbers, including negative numbers. Understand and apply ratio and rate, including percents, and connect ratio and rate to fractions and decimals. Understand and apply proportional reasoning. Understand, estimate, and represent real numbers, including common irrational numbers and with scientific notation.
Content Standards for Grades 6-8	
Content Standard 1: Students can understand and apply a variety of math concepts.	
Benchmark: Students can understand and apply number properties and operations.	
Grade Level Indicator : Represent, compare, and order numbers	
Grade Level Indicator : Describe and apply properties of numbers	
Grade Level Indicator : Classify numbers by divisibility	
Grade Level Indicator : Demonstrate ways of performing operations	
Grade Level Indicator : Use place value; write numbers in standard, expanded, and exponential form	
Grade Level Indicator : Use and interpret operational and relational symbols	
Benchmark: Students can understand and apply concepts and procedures of algebra.	
Grade Level Indicator : Solve equations and inequalities	
Grade Level Indicator : Use variable expressions to model situations	
Grade Level Indicator : Explore numerical patterns	
Benchmark: Students can understand and apply concepts of geometry.	
Grade Level Indicator : Identify, classify, and compare geometric figures	
Grade Level Indicator : Describe geometric properties, patterns, and relationships	
Grade Level Indicator : Apply the concepts of perimeter, area, and volume	
Benchmark: Students can understand and apply concepts of measurement.	
Grade Level Indicator : Measure length/distance, time, temperature, weight, mass, and volume	
Grade Level Indicator : Estimate measurements with appropriate precision	
Grade Level Indicator : Identify and use appropriate units of measurement	
Benchmark: Students can understand and apply concepts in probability and statistics.	
Grade Level Indicator : Apply probability concepts and counting rules	
Grade Level Indicator : Understand and apply measures of central tendency and variability	
Content Standard 2: Students can understand and apply methods of estimation.	
Benchmark: Students can understand and apply concepts and procedures of standard rounding, order of magnitude, and number sense.	
Grade Level Indicator : Use standard rounding to estimate	
Grade Level Indicator : Use order of magnitude to estimate	
Grade Level Indicator : Use number sense to estimate	

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Content Standard 3: Students can solve a variety of math problems.
Benchmark: Students can solve math problems.
Grade Level Indicator : Solve single-step and multiple-step math problems
Grade Level Indicator : Identify extraneous or insufficient information in problems
Benchmark: Students can understand and apply problem-solving approaches and procedures.
Grade Level Indicator : Choose a method for solving a problem
Content Standard 4: Students can interpret data presented in a variety of ways.
Benchmark: Students can use tables and graphs to locate and read information.
Grade Level Indicator : Locate amounts in specific cells of a table
Benchmark: Students can interpret data from a variety of sources.
Grade Level Indicator : Read amounts on scales of bar and line graphs
Grade Level Indicator : Compare quantities to determine ranks, sums, or differences and to find ratios
Grade Level Indicator : Use tables and graphs to determine rates or identify trends, understand underlying or functional relationships, and generalize or draw conclusions
Performance Standards for Grades 6-7
High Performance Level: Understands math concepts, solves word problems, and interprets data from graphs and tables. Usually can use estimation methods. ▪ Distinguished: Understands math concepts, solves word problems, and interprets data from graphs and tables. Usually can use estimation methods. ▪ Accomplished: Understands math concepts, solves word problems, and interprets data from graphs and tables. Usually can use estimation methods. **Intermediate Performance Level:** Usually can understand math concepts, solve word problems, and interpret data from graphs and tables. Sometimes is able to use estimation methods. ▪ Skilled: Usually can understand math concepts, solve word problems, use estimation methods, and interpret data from graphs and tables. ▪ Moderate: Sometimes can understand math concepts and use estimation methods. Usually is able to solve word problems and interpret data from graphs and tables. **Low Performance Level:** Seldom can understand math concepts, solve word problems, or use estimation methods. Sometimes can interpret data from graphs or tables. ▪ Marginal: Sometimes can understand math concepts, use estimation methods, and interpret data from graphs and tables. Seldom is able to solve word problems. ▪ Weak: Seldom can understand math concepts or solve word problems. Rarely can use estimation methods or interpret data from graphs and tables.

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Performance Standards for Grade 8

High Performance Level: A student who performs at this level understands math concepts and is able to solve word problems. The student usually can use estimation methods. The student is able to interpret data from graphs and tables.

- Distinguished: Understands math concepts and is able to solve word problems. Usually can use estimation methods. Is able to interpret data from graphs and tables.
- Accomplished: Understands math concepts and is able to solve word problems. Usually can use estimation methods. Is able to interpret data from graphs and tables.

Intermediate Performance Level: A student who performs at this level usually can understand math concepts and sometimes is able to solve word problems. The student sometimes is able to use estimation methods and usually is able to interpret data from graphs and tables.

- Skilled: Understands math concepts and usually is able to solve word problems. Often can use estimation methods and interpret data from graphs and tables.
- Moderate: Usually can understand math concepts and sometimes is able to solve word problems. Sometimes can use estimation methods and interpret data from graphs and tables.

Low Performance Level: A student who performs at this level seldom can understand math concepts or solve word problems. The student rarely can use estimation methods or interpret data from graphs or tables.

- Marginal: Sometimes can understand math concepts but seldom is able to solve word problems. Sometimes can use estimation methods and interpret data from graphs and tables.
- Weak: Seldom can understand math concepts or solve word problems. Rarely can use estimation methods or interpret data from graphs and tables.

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Iowa Core Curriculum*
K-12 Mathematics

Essential Concepts and Skills with Details and Examples

Introduction

Recent results of national and international tests show that the United States is facing a crisis in mathematics education. American high school students score near the bottom on the international TIMSS and PISA tests, while students in elementary and middle school perform only somewhat better. A common criticism of the U.S. mathematics curriculum is that it is "a mile wide and an inch deep," trying to cover too many topics in not enough depth. All Iowa students must be better prepared in mathematics to successfully compete in the technology-rich, information-dense, global society. To achieve this we must redesign our mathematics curriculum so that it is focused on providing deep understanding of important mathematics.

The Iowa Core Curriculum for K–12 Mathematics identifies the essential characteristics, skills, and content of the world-class mathematics curriculum that Iowa needs. This Iowa Core Curriculum for school mathematics is based on recommendations from the National Council of Teachers of Mathematics (*Principles and Standards for School Mathematics*, 2000, and *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, 2006), five years of experience with Iowa's Every Student Counts mathematics initiative (ESC), and best practices identified from reviews of research conducted by the National Research Council (2001), the International Bureau of Education (Grouws and Cebulla, 2000), the National Council of Teachers of Mathematics (Kilpatrick, 2003), the federal What Works Clearinghouse, and Iowa's Mathematics Content Network research review project.

In addition, the essential skills, concepts, and characteristics recommended here have been informed by a careful review of many background resources, including the *Mathematics Framework for the National Assessment of Educational Progress* (NAEP, 2005 and 2007), *Guidelines for Assessment and Instruction in Statistics Education* (GAISE Report, American Statistical Association, 2005), mathematics standards recommended by Achieve (2007), mathematics standards recommended by the College Board (2007), ACT college readiness standards (2007), the mathematics curricula of Japan and Singapore, the National Center for the Study of Mathematics Curricula, mathematics standards in other states, and recommendations from the Iowa Core Curriculum Project Lead Team. Further resources consulted are included in the Bibliography.

In order to provide effective guidance and technical assistance for Iowa's schools, the Iowa Core Curriculum for K–12 Mathematics draws from the above resources to identify the essential skills, content, and characteristics of a world-class school mathematics curriculum.

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Iowa CORE Curriculum

Primary
(Kindergarten – Grade 2)

Number and Operations

Overall in the number and operations strand, students should "understand numbers, ways of representing numbers, relationships among numbers, and number systems; understand meanings of operations and how they relate to one another; and compute fluently and make reasonable estimates" (NCTM, 2000, p. 32). Specific goals are the following.

Count, represent, read, compare, order and conserve (knows that the total number does not change when configured differently) whole numbers. In particular, students will:

- Count, represent, read, compare, order, and conserve whole numbers up to 1000.
- Write, compare, and order numbers to at least 120 using the words *equal to, greater than, less than, greatest, and least* when appropriate.
- Count by tens or hundreds, forwards and backwards, starting at any number from 1 to 1000.
- Represent numbers to at least 1000 in different way using written words, numerals, or models, and translate among representations.
- Identify the placement and relationships between digits and their values in numbers up to 1000.

Develop understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts. In particular, students will:

- Solve and create story problems that match addition or subtraction expressions or equations using physical objects, pictures, or words.
- Solve simple story problems (result unknown) involving joining, separating, and grouping situations. Solve story problems involving joining, separating, comparing, grouping, and partitioning using a variety of strategies, such as direct modeling with objects or pictures, counting on and counting back, and using related facts and known facts.
- Add and subtract two-digit numbers efficiently and accurately using a procedure that can be generalized, including the standard algorithm and describe why the procedure works.

Express numbers as equivalent representations to fluently compose and decompose numbers (putting together and taking apart). In particular, students will:

- Fluently compose (put together) and decompose (take apart) numbers at least to 10.
- Compose and decompose two- and three-digit numbers based on the values of the digits used to write the number.
- Solve word problems involving joining, separating, part/whole, comparing, grouping, and partitioning, using a variety of strategies, such as direct modeling, counting up or counting back by 1s or 10s, and deriving or recalling facts. (The unknown can appear in a variety of positions).

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Develop fluency and quick recall of addition facts and related subtraction facts and fluency with multi-digit addition and subtraction. In particular, students will:

- Show the inverse relationship between addition and subtraction by using physical models, diagrams, and/or acting-out situations.
- Explain and use strategies for understanding addition facts for sums equal to at least 10, and related subtraction facts.
- Develop and demonstrate quick recall of basic addition facts to 20 and related subtraction facts.
- Solve word problems involving joining, separating, part/whole, comparing, grouping, and partitioning, using a variety of strategies, such as direct modeling, counting up or counting back by 1s or 10s, and deriving or recalling facts. (The unknown can appear in a variety of positions).

Estimate the answer to an addition or subtraction problem before computing, and determine whether the computed answer makes sense. In particular, students will:

- Determine whether the computed answer to an addition or subtraction problem is reasonable.
- Estimate an answer prior to computing. (For example, 23 + 48 is about 70.)

Develop an understanding of whole number relationships, including grouping in tens and ones and apply place-value concepts. In particular, students will:

- Group and count objects by 2s, 5s, and 10s.
- Find a number that is 10 more or 10 less than a given number.
- Group numbers into 10s and 1s in more than one way and explain why the total remains the same.
- Explain and use strategies for remembering addition and subtraction facts to 20.
- Use mental strategies, invented algorithms, and traditional algorithms based on knowledge of place value to add and subtract two-digit numbers.

Understand fractional parts are equal shares or equal portions of a whole unit (a unit can be an object or a collection of things). In particular, students will:

- Understand and represent commonly used fractions, such as 1/4, 1/3, and 1/2.

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Algebra

Overall in the algebra strand, students should "understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts" (NCTM, 2000, p. 37). Specific goals are the following.

Recognize, describe, create and extend repeating and growing patterns such as physical, geometric and numeric patterns and translate from one representation to another. In particular, students will:

- Recognize, describe, create and extend color, rhythmic, shape, number and letter repeating patterns with simple attributes.
- Identify a missing element in a pattern.
- Make a generalization that patterns can translate from one representation to another.
- Recognize, describe, create and extend repeating and growing patterns.
- Translate a pattern between sound, symbols, movements and objects.
- Identify, create, describe, and extend simple number and growing patterns. involving repeated addition and subtraction, skip counting and arrays of objects.
- Use patterns to solve problems in various contexts.

Sort, classify, and order objects by size, number and other properties. In particular, students will:

- Sort and a classify objects by a single attribute and explain the sorting rule.
- Sort and a classify objects by multiple attributes and explain the sorting rule (sort and classify the same set of objects in multiple ways and explain the various sorting rules.).
- Sort and classify a set of objects using a Venn diagram.

Demonstrate the use of the commutative and associative properties and mathematical reasoning to solve for the unknown quantity in addition and subtraction problems; justify the solution. In particular, students will:

- Solve, with objects, simple problems involving joining and separating.
- Develop concepts of addition and subtraction (including commutativity and associativity of addition) using mathematical tools (objects, number line, hundreds chart, etc.), pictures, and mathematical notation.
- Use commutative and associative properties and mathematical reasoning to solve a variety of addition and subtraction problems involving two or more one-digit numbers; justify the solution.

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Understand equality as meaning "the same as" and use the = symbol appropriately. In particular, students will:

- Recognize the use of symbols to represent mathematical ideas in joining and separating problems.
- Determine if equations involving addition and subtraction are true.
- Demonstrate an understanding that the "=" sign means "the same as" by solving open number sentences including those with variables.
- Write number sentences using mathematical notation (+, =, -, <, >, →, and variables) to represent mathematical relationships to solve problems.
- Solve equations in which the unknown and the equal sign appear in a variety of positions.
- Use number sentences involving addition and subtraction, and unknowns to represent and solve given problem situations.

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Geometry

Overall in the geometry strand, students should "analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; specify locations and describe spatial relationships using coordinate geometry and other representational systems; apply transformations and use symmetry to analyze mathematical situations; and use visualization, spatial reasoning, and geometric modeling to solve problems" (NCTM, 2000, p. 41). Specific goals are the following.

Recognize and describe shapes and structures in the physical environment. In particular, students will:

- Identify, name, sort, and describe two- and three-dimensional shapes (including circles, triangles, rectangles, squares, cubes, and spheres), and real-world approximations of the shapes, regardless of size or orientation.

Compose and decompose geometric shapes, including plane and solid figures to develop a foundation for understanding area, volume, fractions, and proportions. In particular, students will:

- Compose (combine) and decompose (take apart) two- and three-dimensional figures and analyze the results.
- Compose and decompose two- and three-dimensional shapes to develop a foundation of fractional relationships and proportions.
- Cover two-dimensional objects with shapes to develop a foundation for area.
- Fill three-dimensional objects to develop a foundation for volume.

Identify, name, sort, and describe two- and three-dimensional geometric figures regardless of size or orientation. In particular, students will:

- Describe characteristics of two- and three-dimensional objects (number of corners, edges, and sides, length of sides, etc.).

Describe and specify space and location with simple relationships and with coordinate systems. In particular, students will:

- Describe the location of one object relative to another object using words such as *in*, *out*, *over*, *under*, *above*, *below*, *between*, *next to*, *behind*, and *in front of*.
- Locate points on maps and simple coordinate grids with letters and numbers.
- Represent points and simple figures on maps using simple coordinate grids with letters and numbers.

Experience and recognize slides, flips, turns and symmetry to analyze mathematical situations. In particular, students will:

- Identify shapes that have been rotated (turned), reflected (flipped), translated (slid), and enlarged. Describe the direction of the translation (left, right, up, down).

Use attributes of geometric figures to solve spatial problems. In particular, students will:

- Describe and represent shapes from different perspective.
- Explore relationships of different attributes.
- Describe geometric shapes in the environment and specify their location.

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Measurement

Overall in the measurement strand, students should "understand measurable attributes of objects and the units, systems, and processes of measurement; and apply appropriate techniques, tools, and formulas to determine measurements" (NCTM, 2000, p. 44). Specific goals are the following.

Identify attributes that are measurable, such as length, weight, time and capacity, and use these attributes to order objects and make direct comparisons. In particular, students will:

- Identify attributes that are measurable such as length, volume, weight, and area. Use these attributes and appropriate language to make direct comparisons. (Taller, shorter, longer, same length; heavier, lighter, same weight; holds more, holds less, holds the same amount).
- Recognize temporal concepts such as before, after, sooner, later, morning, afternoon, evening.
- Use a seriated set of objects to order and compare lengths.
- Recognize that objects used to measure an attribute (length, weight, capacity) must have that attribute and must be consistent in size.
- Determines the relationship between the size of the unit and the number of units needed to make a measurement.

Estimate, measure and compute measurable attributes while solving problems. In particular, students will:

- Select appropriate measurement tools and units (standard and non-standard) to solve problems.

Estimate and measure length using standard (customary and metric) and non-standard units with comprehension. In particular, students will:

- Understand the necessity for identical units (standard or non-standard) for accurate measurements.
- Use a variety of non-standard units to measure length without gaps or overlaps.
- Use non-standard units to compare objects according to their capacities or weights.
- Associate the time of day with everyday events.
- Name standard units of time (day, week, month).
- Use both analog and digital clock to tell time to the hour and half hour.
- Estimate and measure length using metric and customary units.
- Select appropriate measurement tools and units (standard and non-standard) to solve problems.
- Use both analog and digital clock to tell time to the nearest five-minute interval.
- Describe the relationship among standard units of time: minutes, hours days, weeks, months and years.

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Data Analysis and Probability

Overall in the data analysis and probability strand, students should "formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them; select and use appropriate statistical methods to analyze data; develop and evaluate inferences and predictions that are based on data; and understand and apply basic concepts of probability" (NCTM, 2000, p. 48). Specific goals are the following.

Collect, sort, organize, and represent data to ask and answer questions relevant to the K-2 environment. In particular, students will:

- Collect and organize data in lists, tables, and simple graphs.
- Collect, organize, represent, and interpret data in bar-type graphs, picture graphs, frequency tables, and line plots.
- Use interviews, surveys, and observations to collect data that answers questions about themselves and their surroundings.

Compare different representations of the same data using these types of graphs: bar graphs, frequency tables, line plots, and picture graphs. In particular, students will:

- Represent a collection of data using tallies, tables, picture graphs and bar graphs.
- Compare a single data set using different types of graphs.

Use information displayed on graphs to answer questions and make predictions, inferences and generalizations such as likely or unlikely events. In particular, students will:

- Answer simple questions relating to the information displayed on a graph, table, or list.
- Use interviews, surveys, and observations to collect data that answers questions about themselves and their surroundings.
- Analyze information by asking and answering questions about the data.
- Contrast different sets of data displayed on the same type of graph to draw conclusions and make generalizations.
- Use information from data to make observations and inferences, draw conclusions, and make predictions.

Iowa CORE Curriculum

Intermediate
(Grades 3–5)

Number and Operations

Overall in the number and operations strand, students should "understand numbers, ways of representing numbers, relationships among numbers, and number systems; understand meanings of operations and how they relate to one another; and compute fluently and make reasonable estimates" (NCTM, 2000, p. 32). Specific goals are the following.

Develop an understanding of multiplication and division concepts and strategies for basic multiplication facts and related division facts.
In particular, students will:

- Develop concepts of multiplication and division through the use of different representations (e.g. equal-sized groups, arrays, area models, and skip counting on number lines for multiplication, and successive subtraction, partitioning, and sharing for division).
- Use commutative, associative, and distributive properties to develop strategies and generalizations to solve multiplication problems. These strategies will evolve from simple strategies (e.g. times 0, times 1, doubles, count by fives) to more sophisticated strategies, such as splitting the array.
- Relate multiplication and division as inverse operations and learn division facts by relating them to the appropriate multiplication facts.
- Consider the context in which a problem is situated to select the most useful form of the quotient for the solution, and they interpret it appropriately.
- Be able to make comparisons involving multiplication and division, using such words as "twice as many" or "half as many".

Develop fluency and quick recall of multiplication facts and related division facts and fluency with multi-digit multiplication and division.
In particular, students will:

- Extend their work with multiplication and division strategies to develop fluency and recall of multiplication and division facts.
- Apply their understanding of models for multiplication (i.e. equal-sized groups, arrays, area models), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers.
- Apply their understanding of models for division (partitioning, successive subtraction) place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends.
- Develop fluency with efficient procedures for multiplying and dividing whole numbers and use them to solve problems.

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Develop the ability to estimate the results of computation with whole numbers, fractions or decimals and be able to judge reasonableness.
In particular, students will:

- Generalize patterns of multiplying and dividing whole numbers by 10, 100, and 1000 and develop understandings of relative size of numbers.
- Be able to estimate sums and differences with whole numbers up to three digits.
- Build facility and understand when estimation, mental computation or paper-and-pencil computations are appropriate in a given problem.
- Select and apply appropriate strategies (mental computation, number sense and estimation) for estimating products and quotients or determining reasonableness of results, depending on the context and numbers involved.
- Make reasonable estimates of fraction and decimal sums and differences.

Extend place value concepts to represent and compare both whole numbers and decimals.
In particular, students will:

- Extend their understanding of place value to numbers up to 10,000, 100,000 and millions in various contexts and depending on grade level.
- Understand decimal notation as an extension of the base-ten system of writing whole numbers through place-value patterns and models (place-value charts and base-ten blocks) from tenths to hundredths and thousandths, depending on grade level.

Use benchmarks to help develop number sense. In particular, students will:

- Use estimation in determining the relative sizes of number including amounts and distances, such as 500 is 5 flats or 5 x 100, or 500 is ½ of 1000.
- Learn about the position of numbers in the base-ten number system (763 is 7 x 100 plus 6 x 10 plus 3 x 1) and its relationship to benchmarks such as 500, 750, 800 and 1000.
- Extend common benchmarks such as 10, 25, 50, and 100 to understand and use benchmarks of 500 and 1000.
- Understand and use common benchmarks such as ½ or 1 to compare fractions.

Iowa CORE Curriculum

Develop an understanding of commonly used fractions, decimals, and percents, including recognizing and generating equivalent representations.
In particular, students will:

- Develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set, or points or distances on a number line.
- Understand that the size of a fractional part is relative to the size of the whole, and use fractions to represent numbers that are equal to, less than, or greater than 1.
- Solve problems that involve comparing and ordering fractions by using models, benchmark fractions, or strategies involving common numerators or denominators.
- Understand and use models, including the number line, to identify equivalent fractions including numbers greater than one.
- Connect and extend their understanding of fractions to modeling, reading and writing decimals (tenths, hundredth and thousandths), that are greater than or less than 1, identifying equivalent decimals, and comparing and ordering decimals.
- Connect fractions (initially halves, fourths, and tenths, and then fifths, thirds, and eighths) and their equivalent decimals through representations including word names, symbols and models (10 x 10 grids and number lines).
- Recognize and generate equivalent forms of commonly used fractions, decimals and percents.

Develop an understanding of and fluency with addition and subtraction of fractions and decimals. In particular, students will:

- Apply their understandings of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators.
- Apply their understandings of decimal models, place value, and properties to develop strategies to add and subtract fractions and decimals.
- Develop fluency with standard procedures for adding and subtracting fractions and decimals.
- Add and subtract fractions and decimals to solve problems and use number sense to determine reasonableness of results.

Iowa CORE Curriculum

Algebra

Overall in the algebra strand, students should "understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts" (NCTM, 2000, p. 37). Specific goals are the following.

Represent and analyze patterns and relationships involving multiplication and division to introduce multiplicative reasoning.
In particular students will:

- Build a foundation using multiplicative contexts for later understanding of functional relationships with such statements as, "The number of legs is 4 times the number of chairs" or "A quarter is five times the value of a nickel."
- Make generalizations by reasoning about the structure of the pattern to determine if the patterns are nonnumeric growing, repeating, or multiplicative patterns.

Identify the commutative, associative, and distributive properties and use them to compute with whole numbers.
In particular students will:

- Explore the commutative and associative properties through models and examples to determine which properties hold for multiplication and division facts and develop increasingly sophisticated strategies based on these properties and the distributive property to solve multiplication problems involving basic facts.
- Use properties of addition and multiplication to multiply and divide whole numbers and understand why these algorithms work.

Understand and apply the idea of a variable as an unknown quantity and express mathematical relationships using equations.
In particular, students will:

- Use invented notation, standard symbols and variables to express a pattern, generalization, or situation.
- Develop an understanding of the use of a rule to describe a sequence of numbers or objects.
- Use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities.

Represent and analyze patterns and functions, using words, tables, and graphs.
In particular, students will:

- Describe patterns verbally and represent them with tables or symbols.
- Continue to identify, describe, and extend numeric patterns involving all operations and nonnumeric growing or repeating patterns.
- Identify patterns graphically, numerically, or symbolically and use this information to predict how patterns will continue.
- Create graphs of simple equations.
- Be able to use various techniques including words, tables, numbers and symbols for organizing and expressing ideas about relationships and functions.

Iowa CORE Curriculum

Geometry and Measurement

Overall in the geometry and measurement strand, students should "analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; specify locations and describe spatial relationships using coordinate geometry and other representational systems; apply transformations and use symmetry to analyze mathematical situations; and use visualization, spatial reasoning, and geometric modeling to solve problems" (NCTM, 2000, p. 41). Students should also "understand measurable attributes of objects and the units, systems, and processes of measurement; and apply appropriate techniques, tools, and formulas to determine measurements" (NCTM, 2000, p. 44). Specific goals are the following.

Describe, analyze and classify two-dimensional and three-dimensional shapes.
In particular, students will:

- Describe, analyze, and compare two-dimensional shapes by their sides and angles and connect these attributes to definitions of shapes.
- Relate two-dimensional shapes to three-dimensional shapes and analyze properties of polyhedral solids, describing them by the number of edges, faces, or vertices as well as the types of faces.
- Classify two- and three-dimensional shapes according to their attributes and develop definitions of classes of shapes such as parallelograms and prisms.

Explore congruence and similarity. In particular, students will:

- Understand attributes and properties of two-dimensional space through building, drawing and analyzing two-dimensional shapes and use the attributes and properties to solve problems, including applications involving congruence and symmetry.
- Apply congruence to other contexts such as three-dimensional shapes and repeating the congruent shapes to build a similar shape.
- Explore similar shapes to determine that angle measure is the same and the related sides are proportional, that is, related by the same multiplicative or scale factor.

Predict and describe the results of sliding (translation), flipping (reflection), and turning (rotation) two-dimensional shapes. In particular, students will:

- Investigate, describe, and reason about decomposing, combining, and transforming polygons to make other polygons.
- Investigate and describe line and rotational symmetry.
- Extend their understanding of two-dimensional space by using transformations to design and analyze simple tilings and tessellations.

Use ordered pairs on a coordinate grid to describe points or paths (first quadrant).
In particular, students will:

- Learn how to use two numbers to name points on a coordinate grid and know this ordered pair corresponds to a particular point on the grid.
- Make and use coordinate systems to specify locations and to describe paths.
- Explore methods for measuring the distance between two locations on the grid along horizontal and vertical lines.

Iowa CORE Curriculum

Use geometric models to solve problems, such as determining perimeter, area, volume, and surface area. In particular, students will:

- Develop measurement concepts and skills through experiences in analyzing attributes and properties of two- and three-dimensional objects.
- Form an understanding of perimeter as a measurable attribute and quantify perimeter by finding the total distance or length around the shape.
- Recognize area as an attribute of two-dimensional regions and that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps.
- Connect area measure to the area model that has been used to represent multiplication, and use this connection to justify the formula for the area of a rectangle.
- Develop, understand and use formulas to find the area of rectangles, related triangles and parallelograms and learn to measure the necessary attributes of shapes.
- Recognize volume as an attribute of three-dimensional space and understand they can quantify volume by finding the total number of same-sized units of volume that fill the space without gaps or overlaps.
- Decompose three-dimensional shapes to develop strategies for determining surface area.
- Develop strategies to determine the volumes of prisms by layering.

Select and apply appropriate standard (customary and metric) units and tools to measure length, area, volume, weight, time, temperature, and the size of angles.
In particular, students will:

- Select appropriate units, strategies, and tools to solve problems that involve estimating and measuring perimeter, area and volume.
- Develop facility in measuring with fractional parts of linear units.
- Understand that a square that is 1 unit on a side is the standard unit for measuring area.
- Understand that a cube that is 1 unit on an edge is the standard unit for measuring volume.
- Select and apply appropriate units, strategies and tools to solve problems that involve estimating and measuring weight, time and temperature.
- Measure and classify angles.

Select and use benchmarks ($\frac{1}{2}$ inch, 2 liters, 5 pounds, etc.) to estimate measurements. In particular, students will:

- Develop strategies for estimating measurements using appropriate benchmarks, both standard units such as 1 foot and nonstandard units such as the length a book.
- Learn to use strategies involving multiplicative reasoning to estimate measurements (i.e. estimating their teacher's height to be one and a quarter times the student's own height).
- Estimate angle measure using a right angle as the benchmark.

Iowa CORE Curriculum

Data Analysis and Probability

Overall in the data analysis and probability strand, students should "formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them; select and use appropriate statistical methods to analyze data; develop and evaluate inferences and predictions that are based on data; and understand and apply basic concepts of probability" (NCTM, 2000, p. 48). Specific goals are the following.

Represent and analyze data using tallies, pictographs, tables, line plots, bar graphs, circle graphs and line graphs. In particular, students will:

- Recognize the differences representing categorical and numerical data.
- Construct and analyze frequency tables, bar graphs, picture graphs, and line plots and use them to address a question.
- Compare different representations of the same data and evaluate how well each representation shows important aspects of the data.
- Use their understanding of whole numbers, fractions, and decimals to construct and analyze circle graphs and line graphs.
- Apply their understanding of place value to develop and use stem-and-leaf plots.

Describe the distribution of the data using mean, median, mode or range. In particular, students will:

- Learn to compare related data sets, noting the similarities and differences between the two sets and develop the idea of a "average" value.
- Learn to select and use measures of center: mean, median and mode and apply them to describing data sets.
- Build an understanding of what the measures of center tells them about the data and to see this value in the context of other characteristics of the data such as the range.
- Begin to conceptually explore the meaning of mean as the balance point for the data set.

Propose and justify conclusions and predictions based on data. In particular, students will:

- Learn how to describe data, make a prediction to describe the data, and then justify their predictions.
- Learn to collect data using observations, surveys and experiments and propose conjectures.
- Design simple experiments to examine their conjectures and justify their conclusions.
- Design investigations to address a question and consider how data collection methods affect the nature of the data set.
- Examine the role of sample size has in predictions about data.

Iowa CORE Curriculum

Predict the probability of simple experiments and test predictions. In particular, students will:

- Examine the probability of experiments that have only a few outcomes, such as game spinners (i.e., how likely is it that the spinner will land on a particular color?), by first predicting the probability of the desired event and then exploring the outcome through experimental probability.
- Learn to represent the probability of a certain event as 1 and the probability of an impossible event as 0.
- Learn to use common fractions to represent events that are neither certain nor impossible.

Describe events as likely or unlikely and discuss the degree of likelihood using words like certain, equally likely and impossible. In particular, students will:

- Understand probability as the measurement of the likelihood of events.
- Learn to estimate the probability of events as certain, equally likely or impossible by designing simple experiments to collect data and draw conclusions.

Iowa CORE Curriculum

Middle
(Grades 6–8)

Number and Operations

Overall in the number and operations strand, students should "understand numbers, ways of representing numbers, relationships among numbers, and number systems; understand meanings of operations and how they relate to one another; and compute fluently and make reasonable estimates" (NCTM, 2000, p. 32). Specific goals are the following.

Understand, apply, and be computationally fluent with multiplication and division of fractions and decimals. In particular, students will:

- Understand that multiplying two numbers does not necessarily make a bigger number, nor does dividing always result in a smaller number.
- Understand and explain procedures for multiplying and dividing fractions by using the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division.
- Understand and explain procedures for multiplying and dividing decimals by using the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number).
- Use common procedures to multiply and divide fractions and decimals efficiently and accurately.
- Convert from one unit to another in the metric system of measurement by using understanding of the relationships among the units and by multiplying and dividing decimals.
- Convert from one unit to another in the customary system of measurement by using understanding of the relationships among the units and by multiplying and dividing fractions.
- Multiply and divide fractions and decimals to solve problems, including multi-step problems.

Understand, apply, and be computationally fluent with rational numbers, including negative numbers. In particular, students will:

- Understand negative numbers in terms of their position on the number line, their role in the system of all rational numbers, and in everyday situations (e.g., situations of owing money or measuring elevations above and below sea level).
- Understand absolute value in terms of distance on the number line and simplify numerical expressions involving absolute value.
- By applying properties of arithmetic and considering negative numbers in everyday contexts, explain why the rules for adding, subtracting, multiplying, and dividing with negative numbers make sense.
- Understand positive integer exponents in terms of repeated multiplication and evaluate simple exponential expressions.
- Effectively compute with and solve problems using rational numbers, including negative numbers.

Iowa CORE Curriculum

Understand, estimate, and represent real numbers, including common irrational numbers and with scientific notation. In particular, students will:

- Recognize that the set of real numbers, which can be represented as the number line, consists of two disjoint sets – the set of rational numbers and the set of irrational numbers.
- Estimate irrational numbers and represent them as points on the number line.
- Recognize irrational numbers as non-repeating, non-terminating decimals, including common irrational numbers such as π and non-perfect square roots and cube roots
- Understand and determine the square roots of perfect squares.
- Understand and estimate square roots of non-perfect-squares, and determine more precise values using a calculator.
- Represent, use, and interpret numbers in scientific notation.
- Use scientific notation and rational and irrational numbers to model and solve problems.

Understand and apply ratio and rate, including percents, and connect ratio and rate to fractions and decimals. In particular, students will:

- Build on understanding of fractions and part-whole relationships to understand ratios (by, for example, analyzing the relative quantities of boys and girls in the classroom or triangles and squares in a drawing).
- Understand percent as a rate and develop fluency in converting among fractions, decimals, and percents.
- Understand equivalent ratios as deriving from, and extending, pairs of rows (or columns) in the multiplication table.
- Understand rate as a way to compare unlike quantities (such as miles per hour or a situation in which 5 pens cost $3.75).
- Use a variety of strategies to solve problems involving ratio and rate.

Understand and apply proportional reasoning. In particular, students will:

- Understand that a proportion is an equation that states that two ratios are equivalent.
- Understand proportional relationships ($y = kx$ or $\frac{y}{x} = k$), and distinguish proportional relationships from other relationships, including inverse proportionality ($xy = k$ or $y = \frac{k}{x}$).
- Understand that in a proportional relationship of two variables, if one variable doubles or triples, for example, then the other variable also doubles or triples, and if one variable changes additively by a specific amount, *a*, then the other variable changes additively by the amount *ka.*
- Graph proportional relationships and identify the constant of proportionality as the slope of the related line.
- Use ratios and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease.
- Use proportionality to solve single and multi-step problems in a variety of other contexts.

Iowa CORE Curriculum

Algebra

Overall in the algebra strand, students should "understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts" (NCTM, 2000, p. 37). Specific goals are the following.

Write, interpret, and use mathematical expressions and equations, find equivalent forms, and relate such symbolic representations to verbal, graphical, and tabular representations. In particular, students will:

- Write mathematical expressions, equations, and formulas that correspond to given situations.
- Understand that variables represent numbers whose exact values are not yet specified, use single letters, words, or phrases as variables, and use variables appropriately.
- Evaluate expressions (for example, find the value of $3x$ if x is 7).
- Understand that expressions in different forms can be equivalent, and rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information).
- Understand that solutions of an equation are the values of the variables that make the equation true.
- Solve simple one-step equations (i.e., involving a single operation) by using number sense, properties of operation, and the idea of maintaining equality on both sides of an equation.
- Construct and analyze tables (e.g., to show quantities that are in equivalent ratios), and use equations to describe simple relationships shown in a table (such as $3x = y$).
- Use expressions, equations, and formulas to solve problems, and justify their solutions.

Understand and apply proportionality. In particular, students will:

- Understand that a proportion is an equation that states that two ratios are equivalent.
- Understand proportional relationships ($y = kx$ *or* $\frac{y}{x} = k$), and distinguish proportional relationships from other relationships, including inverse proportionality ($xy = k$ *or* $y = \frac{k}{x}$).
- Graph proportional relationships and identify the constant of proportionality as the slope of the related line.
- Use ratios and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease.
- Use proportionality to solve single and multi-step problems in a variety of other contexts.

Iowa CORE Curriculum

Understand, solve, and apply linear equations and inequalities. In particular, students will:

- Make strategic choices of procedures to solve linear equations and inequalities in one variable and implement them efficiently.
- Recognize and generate equivalent forms of linear expressions, by using the associative, commutative, and distributive properties.
- Understand that when properties of equality are used to transform an equation into a new equivalent equation, solutions obtained for the new equation also solve the original equation.
- Solve more complicated linear equations, including solving for one variable in terms of another.
- Solve linear inequalities and represent the solution on a number line.
- Formulate linear equations and inequalities in one variable and use them to solve problems, including in applied settings, and justify the solution using multiple representations.

Understand and apply linear functions. In particular, students will:

- Understand linear functions and slope of lines in terms of constant rate of change.
- Understand that the slope of a line is constant, for example by using similar triangles (e.g., as shown in the rise and run of "slope triangles"), and compute the slope of a line using any two points on the line.
- Build on the concept of proportion, recognizing a proportional relationship ($\frac{y}{x} = k$, or $y = kx$) as a special case of a linear function. In this special case, understand that if one variable doubles or triples, for example, then the other variable also doubles or triples; and understand that if the input, or x-coordinate in this case, changes additively by a specific amount, a, then the output, or y-coordinate in this case, changes additively by the amount ka.
- Understand that the graph of the equation $y = mx + b$ is a line with y-intercept b and slope m.
- Translate among verbal, tabular, graphical, and algebraic representations of functions, including recursive representations such as NEXT = NOW +3 (recognizing that tabular and graphical representations often only yield approximate solutions), and describe how such aspects of a linear function as slope, constant rate of change, and intercepts appear in different representations.
- Use linear functions, and understanding of the slope of a line and constant rate of change, to analyze situations and solve problems.

Use tables and graphs to analyze systems of linear equations. In particular, students will:

- Use tables and graphs to analyze and (approximately) solve systems of two linear equations in two variables.
- Relate a system of two linear equations in two variables to a pair of lines in the plane that intersect, are parallel, or are the same.
- Use systems of linear equations to analyze situations and solve problems.

Iowa CORE Curriculum

Geometry

Overall in the geometry strand, students should "analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; specify locations and describe spatial relationships using coordinate geometry and other representational systems; apply transformations and use symmetry to analyze mathematical situations; and use visualization, spatial reasoning, and geometric modeling to solve problems" (NCTM, 2000, p. 41). Specific goals are the following.

Understand, determine, and apply area of polygons. In particular, students will:

- Use physical models, such as geoboards, to develop and make sense of area formulas.
- Use knowledge of area of simpler shapes to help find area of more complex shapes.
- Understand and apply formulas to find area of triangles and quadrilaterals.
- Solve problems related to and using area, including in real-world settings.

Understand and apply similarity, with connections to proportion. In particular, students will:

- Understand that two objects are similar if they have the same shape (i.e., corresponding angles are congruent) but not necessarily the same size.
- Understand similarity in terms of a scale factor between corresponding lengths in similar objects (i.e., similar objects are related by transformations of magnifying or shrinking).
- Understand that relationships of lengths within similar objects are preserved (i.e., ratios of corresponding sides in similar objects are equal).
- Understand that congruent figures are similar with a scale factor of 1.
- Use understanding of similarity to solve problems in a variety of contexts.

Understand, determine, and apply surface area and volume of prisms and cylinders and circumference and area of circles. In particular, students will:

- Find the area of more complex two-dimensional shapes, such as pentagons, hexagons, or irregular shaped regions, by decomposing the complex shapes into simpler shapes, such as triangles.
- Understand that the ratio of the circumference to the diameter of a circle is constant and equal to π, and use this fact to develop a formula for the circumference of a circle.
- Understand that the formula for the area of a circle is plausible by decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram.
- Develop and justify strategies for determining the surface area of prisms and cylinders by determining the areas of shapes that comprise the surface.
- By decomposing prisms and cylinders by slicing them, develop and understand formulas for their volumes (Volume = Area of base x Height).
- Select appropriate two-and three-dimensional shapes to model real-world situations and solve a variety of problems (including multi-step problems) involving surface area, area and circumference of circles, and volume of prisms and cylinders.

Iowa CORE Curriculum

Analyze two-dimensional space and figures by using distance, angle, coordinates, and transformations. In particular, students will:

- Explore and explain the relationships among angles when a transversal cuts parallel lines.
- Use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and apply this fact about triangles to find unknown measures of angles.
- Understand and explain how particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines (e.g., "slope triangles").
- Use reasoning about similar triangles to solve a variety of problems, including those that involve determining heights and distances.
- Explain why the Pythagorean Theorem is valid by using a variety of methods – for example, by decomposing a square in different ways.
- Apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane and to measure lengths and analyze polygons.
- Understand and apply transformations – reflection, translation, rotation, and dilation, and understand similarity, congruence, and symmetry in terms of transformations.

Visualize, represent, and describe three-dimensional shapes. In particular, students will:

- Recognize and draw two-dimensional representations of three-dimensional figures, including nets, front-side-top views, and perspective drawings.
- Identify and describe three-dimensional shapes, including prisms, pyramids, cylinders, and spheres.
- Examine, build, compose, and decompose three-dimensional objects, using a variety of tools, including paper-and-pencil, geometric models, and dynamic geometry software.
- Use visualization and three-dimensional shapes to solve problems, especially in real-world settings.

Iowa CORE Curriculum

Data Analysis and Probability

Overall in the data analysis and probability strand, students should "formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them; select and use appropriate statistical methods to analyze data; develop and evaluate inferences and predictions that are based on data; and understand and apply basic concepts of probability" (NCTM, 2000, p. 48). Specific goals are the following.

Understand, interpret, determine, and apply measures of center and graphical representations of data. In particular, students will:

- Extend prior work with mode, median, and mean as measures of center.
- Compute the mean for small data sets and explore its meaning as a balance point for a data set.
- Extend prior work with bar graphs, line graphs, line plots, histograms, circle graphs, and stem and leaf plots as graphical representations of data to include box-and-whisker plots and scatterplots.
- Create and interpret box-and-whisker plots and scatterplots.

Analyze and summarize data sets, including initial analysis of variability. In particular, students will:

- Select, determine, explain, and interpret appropriate measures of center for given data sets (mean, median, mode).
- Select, create, explain, and interpret appropriate graphical representations for given data sets (bar graphs, circle graphs, line graphs, histograms, line plots, stem and leaf plots, box-and-whisker plots, scatterplots).
- Summarize and compare data sets using appropriate numerical statistics and graphical representations.
- Compare the information provided by the mean and the median and investigate the different effects that changes in the data values have on these measures of center.
- Understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center, and thus consider and describe the variability of the data (e.g, range and interquartile range).
- Informally determine a line of best fit for a scatterplot to make predictions and estimates.
- Formulate questions, gather data relevant to the questions, organize and analyze the data to help answer the questions, including informal analysis of randomness and bias.

Iowa CORE Curriculum

Use proportions and percentages to analyze data and chance. In particular, students will:

- Use proportions to make estimates relating to a population on the basis of a sample.
- Apply percentages to make and interpret histograms and circle graphs.
- Explore situations in which all outcomes of an experiment are equally likely, and thus the theoretical probability of an event is the number of outcomes corresponding to the event divided by total number of possible outcomes.
- Use theoretical probability and proportions to make approximate predictions.

Understand and represent simple probabilistic situations. In particular, students will:

- Represent the probability of events that are impossible, unlikely, likely, and certain using rational numbers from 0 to 1.
- List all possible outcomes of a given experiment or event.

Understand, compute, and estimate simple probabilities using counting strategies and simulation. In particular, students will:

- Understand and apply the Multiplication Principle of Counting in simple situations.
- Compute probabilities for compound events, using such methods as organized lists, tree diagrams (counting trees), area models, and counting principles.
- Estimate the probability of simple and compound events through experimentation and simulation.
- Use a variety of experiments to explore the relationship between experimental and theoretical probabilities and the effect of sample size on this relationship.

Iowa CORE Curriculum

Primary
(Kindergarten–Grade 2)

Rigor and Relevance Framework **Grade Band: K–2** **Strand:** **Number and Operations** **Essential Concept:** **Develop understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts**	
Quadrant C Students solve this problem: I have six crayons and my friend gave me some more and now I have ten. How many crayons did my friend give me?	**Quadrant D** Students solve this problem: I opened a box of ten crayons. Each of the crayons in the box is either red or blue. What are the possible combinations of red and blue crayons that could be in my box?
Quadrant A Students solve this problem: $6 + 4 = \square$	**Quadrant B** Students solve this problem: I have six crayons and my friend gave me four more. How many crayons do I have now?

Iowa CORE Curriculum

Rigor and Relevance Framework	
Grade Band:	**K–2**
Strand:	**Algebra**
Essential Concept:	**Recognize, describe, and extend repeating and growing patterns such as physical, geometric and numeric patterns and translate from one representation to another**

Quadrant C Students are asked to create an ABAB pattern. Pattern blocks are available for use as needed. Ask students to describe the pattern and name the next three components.	**Quadrant D** In pairs, each student creates a repeating linear pattern for a table runner using a set of pattern blocks. Students ask their partner to tell them the next shape in their pattern, then the next. Students ask their partner to predict the fifteenth shape in the pattern. Students duplicate the pattern to produce the table runner using appropriate materials.
Quadrant A Using pattern blocks, the teacher creates an ABAB pattern. The students are requested to duplicate the teacher's pattern.	**Quadrant B** In art class, students use pattern blocks of only one shape and three different color (or 3 shapes of one color) and design their own pattern.

Iowa CORE Curriculum

Rigor and Relevance Framework	
Grade Band:	***K–2***
Strand:	***Geometry***
Essential Concept:	***Compose and decompose geometric shapes, including plane and solid figures, to develop a foundation for understanding area, volume, fractions, and proportions***

Quadrant C Students are given a set of pattern blocks with all squares and tan rhombuses removed. Teacher asks students to show: 1) How many green triangles are needed to cover one blue rhombus? 2) How many green triangles to cover three blue rhombuses? 3) How many green triangles do you need to cover one red trapezoid? 4) What other relations can you show between the shapes?	***Quadrant D*** During the fish unit, children are given two outlines, one of a long skinny fish and one of a short fat fish. Given a set of pattern blocks with all squares and tan rhombuses removed, students are asked to cover the shapes and determine which fish has the larger area.
Quadrant A Using a set of pattern blocks with all squares and tan rhombuses removed, show the students different shapes and ask them to name the shapes. Teacher asks student to show how many green triangles are needed to cover a blue rhombus.	***Quadrant B*** Students design the background for the cover of a shapes book. The cover has connected outlines of the hexagon shapes. Students are provided hexagons, triangles, trapezoids, and rhombuses to create the cover.

Iowa CORE Curriculum

Rigor and Relevance Framework	
Grade Band:	***K–2***
Strand:	***Measurement***
Essential Concept:	***Estimate and measure length using standard (customary and metric) and non-standard units with comprehension***

Quadrant C Using unifix cubes, work with a partner, measure your arm span and your height. What do you and your partner notice about the measurements?	***Quadrant D*** Students solve this problem: The library is getting new bookshelves. Your teacher would like to have one of the short bookshelves in your classroom, but she doesn't know if one will fit. The only possible place the bookshelf will fit is under the window. You and your partner decide on a unit of measure to use when determining if the bookshelf will fit. Record what you did and what your results were.
Quadrant A Using unifix cubes, measure the length of the lines on the worksheet your teacher has given you. Record your results.	***Quadrant B*** Using unifix cubes, measure the length and width of your math book. Record your results.

Iowa CORE Curriculum

Rigor and Relevance Framework **Grade Band:** **K–2** **Strand:** **Data Analysis and Probability** **Essential Concept:** **Collect, sort, organize, and represent data to answer questions relevant to the K–2 environment**	
Quadrant C Given a set of buttons, students are asked to sort them by an attribute, determine how many are in each group, and order the groups from least to greatest.	**Quadrant D** Students generate a question they want answered by the class, collect the data, represent the information on a graph and report the findings to the class.
Quadrant A Given a set of buttons, students are asked to sort them by color and determine the quantity of each group.	**Quadrant B** Each student makes a unifix train with one cube for each button they have on their clothing. The class organizes their individual unifix trains from least to greatest to create a class graph.

Iowa CORE Curriculum

Intermediate
(Grades 3–5)

Rigor and Relevance Framework **Grade Band:** **3–5** **Strand:** **Number and Operations** **Essential Concept:** **Develop an understanding of multiplication and division concepts and strategies for basic multiplication facts and related division facts**	
Quadrant C Use base ten blocks to show 24 × 8 and explain your work.	**Quadrant D** Write a story problem to illustrate 24 × 8 and show at least two ways to get the answer.
Quadrant A Multiply: 24 × 8	**Quadrant B** Draw a diagram and explain how you would find the area of a room that measures 24 feet by 8 feet.

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Rigor and Relevance Framework **Grade Band:** **3–5** **Strand:** **Algebra** **Essential Concept:** **Understand and apply the idea of variable as an unknown quantity and express mathematical relationships using equations**	
Quadrant C Using cubes illustrate the following relationships: One blue cube plus two yellow cubes equal 8 red cubes. Two yellow cubes equal 3 red cubes. How many red cubes equal one blue cube?	**Quadrant D** After reading the story of "Acrobats, Grandmas and Ivan" write an equation in which the Grandmas win without Ivan. Explain why this is or is not possible.
Quadrant A Solve: X + 2Y = 8Z 2Y = 3Z X = ? Z	**Quadrant B** Solve the following problem and explain your work: Ivan and 2 acrobats were tied with 8 Grandmas in the first round of a tug-of-war. Then in the second round 2 acrobats tied with 3 Grandmas. What would happen in the third round with 5 Grandmas against Ivan?

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Rigor and Relevance Framework	
Grade Band:	**3–5**
Strand:	**Geometry and Measurement**
Essential Concept:	**Describe, analyze, and classify two-dimensional and three-dimensional shapes**
Quadrant C Draw as many different nets, as possible, for cubes that are 3 cm on each side. • Are any of the nets identical? How can you tell? • Without folding, can you determine if a net will fold into a cube? How? • What properties are common to all nets that will form a cube?	**Quadrant D** A box company wants to save money, so they try to fit as many nets as possible on one sheet. If the company uses a cardboard shape that is 20 cm by 20 cm, how many nets of any type will fit? They can be arranged in any way as long as the net folds into a cube.
Quadrant A Looking at the drawing of a cube in a textbook, list the properties or characteristics of the cube.	**Quadrant B** A box company needs a cube that is 3 cm on each side for jewelry boxes. How many different nets can you draw that can be folded into a cube that is 3 cm on each side?

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Rigor and Relevance Framework **Grade Band:** **3–5** **Strand:** **Data Analysis and Probability** **Essential Concept:** **Represent and analyze data using tallies, pictographs, tables, line plots, bar graphs, circle graphs, and line graphs**	
Quadrant C Represent data found in a textbook or on the web, in a line plot. Have students describe where the mean, median and mode are on the line plot. Are they the same or different? Why? How does the shape of the data relate to the mean, median and mode?	**Quadrant D** Ask students to predict how long they can stand on one foot, with their eyes closed. Will it be different standing on their right and left foot? Have students discuss the appropriate guidelines for collecting this data. Have students work in pairs and collect the data and display it on two line plots for the class. Ask students to write an analysis of the data using mean, median or mode. Ask them to describe the shape of the data and what it means. Ask students to compare their predictions to the actual data.
Quadrant A Represent data found in a textbook or on the web in a line plot. Using the textbook definition of mean, show where the mean is on the line plot.	**Quadrant B** Working in pairs, ask students to collect data on how long they can stand on one foot with their eyes closed. Display this data in two line plots for the class. Ask students to identify the mean, median and mode of the data.

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Middle
(Grades 6–8)

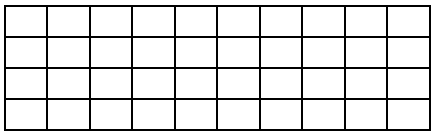
Rigor and Relevance Framework

Grade Band: **6–8**
Strand: **Number and Operations**
Essential Concept: **Understand, apply, and be computationally fluent with multiplication and division of fractions and decimals**
Essential Concept: **Understand and apply ratio and rate, including percents, and connect ratio and rate to fractions and decimals**

Quadrant C

Shade 6 of the small squares in the rectangle shown below.

Using the



rectangle, explain how to determine each of the following:

- the percent of the area that is shaded
- the decimal part of the area that is shaded
- the fractional part of the area that is shaded

See Lesson Plan in Appendix C.

(Adapted from Ron Castleman's Task in the QUASAR Project (*Implementing Standards-Based Mathematics Instruction*, Stein, Smith, Henningsen, Silver, Teachers College Press, 2000; and *Navigating Through Number and Operations in Grades 6–8*, NCTM, pp. 26–28)

Quadrant D

Divide students into teams with sets of magazines and newspapers. Each team should do the following. Find examples of fractions, decimals, and percents in ads and articles. Show where each of these are being used. Indicate when it would be appropriate to use the other forms, then state what those numbers would be. Create problems related to the ads that would be appropriate to solve using fractions, decimals, and percents. Solve the problems. Create a presentation to share with the class that displays, summarizes, and explains your work.

Quadrant A

Shade 10 X 10 grids – then give the appropriate fraction, decimal, and percent representations.

Shade 50 squares
Fraction ___ Decimal ___ Percent ___
Shade 20 squares
Fraction ___ Decimal ___ Percent ___
Shade 10 squares
Fraction ___ Decimal ___ Percent ___
Shade 1 square
Fraction ___ Decimal ___ Percent ___
Shade 150 squares
Fraction ___ Decimal ___ Percent ___

Quadrant B

Give the students a newspaper with a set of ads showing various percent-off sales. Given several 10 x 10 grids, students color in a grid for each ad showing the percent. Also label the colored part of the grid with the related fractions and decimals.

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Rigor and Relevance Framework **Grade Band:** **6–8** **Strand:** **Algebra** **Essential Concept:** **Write, interpret, and use mathematical expressions and equations, find equivalent forms, and relate such symbolic representations to verbal, graphical, and tabular representations**	
Quadrant C Given a table of values for two variables: • Look for all patterns that can be found in the table. What patterns did you find? • What recursive formula(s) did you find? • What direct (explicit) formulas did you find? • What are some advantages and disadvantages of the two types of formulas? • Are the formulas equivalent? How do you know? • Why are equivalent formulas useful?	**Quadrant D** Examine metal beams in pictures of a construction site. If the length of the beam is determined by the number of rungs on the underside of the beam, determine how many rods are needed to make different lengths of beams. • Make a table of this information. • Describe any patterns you see in the table. • What recursive formula(s) can you find? • What direct (explicit) formulas can you find? • What are some advantages and disadvantages of the two types of formulas? • Are the formulas equivalent? How do you know? • Why are equivalent formulas useful? Explain (justify) the formulas in the context of beams and rods. What generalizations can you make about how many rods will be needed for any beam length? (Adapted from Modeling Middle School Mathematics – MMM Project – lesson and classroom video: http://mmmproject.org/vp/mainframeS.htm)

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Quadrant A

Fill in the following table.
What patterns do you see?

1	3
2	7
3	11
4	
	27
8	
	39

Quadrant B

In order to build a trestle for your model, you will use toothpicks to build the beams.

- Use 3 toothpicks to create a beam of length 1.
- Make beams of length 2, 3, 4, 5 and 6.
- How many toothpicks are used to make each beam?
- Make a table with the length of the beam on one side and the number of toothpicks on the other side.
- Find any patterns you can in this table.
- Without building, determine how many toothpicks would be needed to build a beam of length 7, 8, 9 and 10.

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Rigor and Relevance Framework ***Grade Band:*** ***6–8*** ***Strand:*** ***Geometry*** ***Essential Concept:*** ***Understand, determine, and apply surface area and volumes of prisms and cylinders and circumference and area of circles***	
Quadrant C The task is to investigate how changing the lengths of the sides of a rectangular prism affects the volume and surface area of the prism. Students are given an applet (software) that shows two rectangular prisms that are congruent (equal angles and equal sides). Change the size of the second prism. Are the two prisms still congruent? Are they similar? Find the volume using the button "Show Volume." Change the size of the second prism again and observe the changes in the measurements. What is being depicted in the graphs? What can you say about the relationship between the side lengths and the volume of a rectangular prism? About the side lengths and the surface area? (Adapted from NCTM *Principles and Standards* E-example: http://standards.nctm.org/document/eexamples/chap6/6.3/part2.htm)	**Quadrant D** Emma works at the Acme Box Factory. Her job is to construct cubes that will be used as jewelry boxes. Her job is to find as many unique nets for boxes that are 3 cm per side as she can. Find all the different nets that can be folded into a cube. Describe the properties of a cube and its nets, and explain how you know you have found all possible nets that will form a cube. The company wants to make these jewelry boxes as efficiently as possible. They can save money by fitting as many nets as possible on one piece of cardboard. The company will be using cardboard that is 20 cm by 20 cm. What is the greatest number of nets (of any type) that can be arranged to fit on one piece of cardboard? (Adapted from NCTM Illuminations: *http://illuminations.nctm.org/LessonDetail.aspx?id=L570)*
Quadrant A A box has dimensions of 60 cm, 18 cm, and 8 cm. What is the volume of the box? What is the surface area of the box?	**Quadrant B** My safety deposit box has dimensions of 60 cm, 18 cm, and 8 cm. How many $20 bills can I fit in it? How much money can it hold? How much paint would it take to paint the outside of the safety deposit box? (Adapted from http://www.nsa.gov/teachers/ms/geom20.pdf)

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Rigor and Relevance Framework	
Grade Band:	**6–8**
Strand:	**Data Analysis and Probability**
Essential Concept:	**Understand, interpret, determine, and apply measures of center and graphical representations of data**
Essential Concept:	**Analyze and summarize data sets, including initial analysis of variability**

Quadrant C	**Quadrant D**
Are students carrying backpacks that are too heavy? Read an article about this issue. Examine *TinkerPlots* data already collected from a group of 1st, 5th, and 7th grade students regarding their weight and the weight of their backpacks. (**See Appendix C**). Create new sets of data showing what percent backpack weight is of student weight. Find the mean, median, and mode of these new sets of data and describe the variability. Graphically represent the data. Draw conclusions about whether students or groups of students are carrying backpacks that are too heavy. Provide evidence to support your conclusions. (Adapted from *Tinkerplots* activity, Key Curriculum Press, http://www.keypress.com/x5715.xml)	A group of students will design and conduct a statistical study to answer a question they have formulated (for example, a question about backpack weight). They can either collect data or use sources of information containing the data needed to answer the question. They will analyze and summarize the data, including measures of center and variability, and graphically represent the data. They will answer the question they formulated based on the data and their analysis. They will create a brief presentation with visuals and careful explanations.

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Quadrant A

Here is a collection of numbers. Find what percent B is of A. Find the mean, median, and mode of this new set of data and also describe the variability.

Weight A	Weight B
87	21
94	5
78	14
82	12
72	9
114	22
98	19
107	39
120	20
104	27
79	19
95	19

Quadrant B

Examine the data below from a group of seventh graders and their backpacks. Find what percent the backpack weight is of each student's weight. Find the mean, median, and mode of this new set of data and also describe the variability.

Name	Gender	Weight	Backpack Weight
Katie	F	87	21
Deborah	F	94	5
Jennifer	F	78	14
Lori	F	82	12
Sherry	F	72	9
Kathy	F	114	22
Pat	F	98	19
Gayle	F	107	39
Myrle	F	120	20
Jeffrey	M	104	27
Alan	M	79	19
Paul	M	95	19

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