DOINGWHATWORKS

SAMPLE MATERIAL

Concrete-Representational-Abstract (CRA) Instructional Approach Summary Report

The Access Center: American Institutes for Research (AIR), Washington, DC

Topic: Response to Intervention in Elementary-Middle Math

The Concrete-Representational-Abstract (CRA) Instructional Approach provides a graduated, conceptually supported framework to create meaningful connections among concrete, representational, and abstract levels of understanding. Students establish understanding through visual, tactile, and kinesthetic experiences and expand their understanding through pictorial representations before moving to abstract concepts.

Across all grade levels, CRA may be implemented individually, in small groups, or for the entire class. To help students achieve mastery of math concepts in the CRA approach, teachers should provide students multiple opportunities for practice and observation.

This summary report, available from The Access Center website, describes the CRA instructional strategy, shows examples of its use in teaching fractions, includes research references, and suggests resources for further information.

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The Access Center
Improving Outcomes for All Students K-8

Concrete-Representational-Abstract Instructional Approach

What Is the Concrete-Representational-Abstract (CRA) Instructional Approach?

CRA is an intervention for mathematics instruction that research suggests can enhance the mathematics performance of students with learning disabilities. It is a three-part instructional strategy, with each part building on the previous instruction to promote student learning and retention and to address conceptual knowledge.

The CRA instructional sequence consists of three stages: concrete, representation, and abstract:

- *Concrete.* In the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials (e.g., red and yellow chips, cubes, base-ten blocks, pattern blocks, fraction bars, and geometric figures).
- *Representational.* In this stage, the teacher transforms the concrete model into a representational (semiconcrete) level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting.
- *Abstract.* At this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, –, ×, ÷) to indicate addition, multiplication, or division.

Concrete. The "doing" stage using concrete objects to model problems

Representational. The "seeing" stage using representations of the objects to model problems

Abstract. The "symbolic" stage using abstract symbols to model problems

CRA supports understanding underlying mathematical concepts before learning "rules," that is, moving from a concrete model of chips or blocks for multiplication to an abstract representation such as 4 x 3 = 12.

Research-based studies show that students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations (Harrison & Harrison, 1986; Suydam & Higgins, 1977). Some mathematical concepts for which structured concrete materials work well as a foundation to develop understanding of concepts are early number relations, place value, computation, fractions, decimals, measurement, geometry, money, percentage, number bases, word problems, probability and statistics.

What Does CRA Look Like?

The CRA sequence of instruction provides a graduated and conceptually supported framework for students to create a meaningful connection among concrete, representational, and abstract levels of understanding. Beginning with visual, tactile, and kinesthetic experiences to establish understanding, students expand their understanding through pictorial representations of concrete objects and move to the abstract level of understanding.

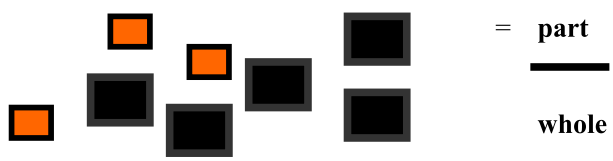
Examples of the mathematical concepts are illustrated below.

Reading and Writing Fractions

Once fraction concepts are understood as "part of a whole," students can practice the steps involved in reading and writing fractions. A variety of physical materials can be used to show the meaning of a fraction as "part of a whole." For example, fraction cubes, counters, fraction bars, or geometric shapes can indicate a fraction (e.g., 3 red cubes (part) out of the 5 cubes (whole, the total number of cubes). Representations and numeric symbols of the fraction can develop the skills of reading and writing fractions. The abstract stage is developed by writing a numeric symbol of the number of squares or parts of the whole in correct fraction form. This step involves the order in which digits should be read or written. For a fraction, which number is (represented) written on the top? Which number is (represented) written on the bottom? An example of reading and writing fractions in the correct order follows.

Reading and Writing Fractions

Goal: To develop the spatial organization, visually and kinesthetically, to read and write fractions correctly.
Materials: Red squares and larger black squares are displayed to help with sequencing and number placement.



Teacher: "Today we are going to write and say fractions."
Concrete: Teacher points to the squares arranged on a table. "What colors are the squares?" (Student says black and red.) "Count the total number of squares (whole)." (Student points and counts to 8. Student says 8.) "How many red squares are there?" (Student points and counts to 3. Student says 3.)
Represent: "When we talk about fractions, we say the 'part of the whole.' (Say together 'part of the whole'). We can write a fraction showing the part of the whole, as shown above. The number for the part is written on the top and the number for the whole is written on the bottom. (Say part on top and whole on bottom.) What was the total number of squares?" (Student says 8.) "Let's call that the whole."
Abstract: "Write the total number of squares or the whole on the bottom where the word 'whole' is shown." (Student writes 8.)
Represent: "How many red squares are there?" (Student says 3.) "Let's call the red squares 'part' of the whole."
Abstract: "Write the number of red squares on the top where the word 'part' is written. (Student writes 3.)
Summary: "From this example, what did you write for the fraction?" (Student says 3 and 8.) "We say 3 out of 8 or three-eighths." Practice several different examples with the squares, writing and reading the fractions.

Adapted from *Teaching Mathematics to Students with Learning Disabilities,* Bley & Thornton, p. 296.

Children also may have difficulty interpreting written decimals and correctly attaching *tenth, hundredth,* and other decimal names. A meaningful way to help the student become functionally independent with writing decimals is to present decimals with visual aids and relate them to the familiar fractional notation.

The following example illustrates a technique for helping students with learning disabilities read and write decimals by relating them to fractions. Children should have a firm understanding of fraction as part of a whole and be able to read and write fractions. In addition, children should have previous experience working with blocks and graph paper.

Goal: To read and write decimals (initial understanding of decimal place value)

Materials: Three blocks representing 1 whole (hundreds block), 1 tenth (tens block), 1 hundredth (ones block), and paper to write on.

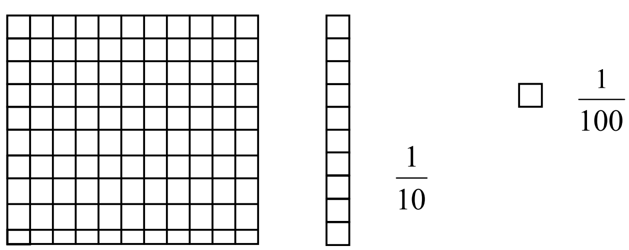
Teacher: "Today we will be working with blocks to read and write decimals." (Place the three blocks in front of student. Help student say and write the correct symbol for each. You may refer to the color cueing in the previous example to help student write the correct notation.)

"Here is the hundreds block. It is one whole block divided into 100 parts. Please write a 1 under this block."

"Here is another block. Does this match one side of the whole block? (Yes) How many parts is this block divided into? (10) We call this block 'one-tenth.' Can you say 'one-tenth'? Now write $\frac{1}{10}$ under this block as a fraction." (Teacher should write the words under the fractions for children with auditory discrimination deficits to distinguish the difference between ten and tenths. Color cueing the "ths" may also help student visually note the difference.)

Stages of CRA

Concrete:



one   one-tenth   one-hundredth

Practice with several blocks, saying the meaning and writing the fractions. Example: Show 1 hundreds block, 2 tens blocks and 3 ones blocks. Child should say and write the correct fraction. The child shows the model that represents the fraction. Example: write: $\frac{4}{10}$ and $\frac{5}{100}$. The child should say and show you the correct number of blocks.

"The last block looks like a cube. Is it part of the whole block also? (Yes) How many parts was the whole divided into? (100) So we call this one part of a hundred 'one hundredth.' Can you say 'one-hundredth'? Now write $\frac{1}{100}$ under the cube as a fraction."

Representation: Teacher can introduce a hundreds grid. Have the student color one tenth and one hundredth. (Teachers may skip this level if they are confident the child sees the connection between the concrete and the fraction notation.)

Abstract: Teacher shows the one-tenth block again and points to the one zero. "One-tenth has one zero. Let's write a decimal for this block and this fraction." Teacher cues the child and writes "One zero, one decimal place .1." Teacher shows the one-hundredth block and points to two zeros. Teacher cues the child and writes "One hundredth has two zeros. Two zeros, count two decimal places and write a one, .01." Teacher should continue practicing with several examples until the child connects the concrete blocks, the number of zeros in the fraction, and the number of decimal places.

$$\frac{1}{10} \quad .1 \qquad\qquad \frac{1}{100} \quad .01$$

Example: Show 1 hundreds block, 3 tenths blocks, and 4 one-hundredths blocks. Child can use a guide at first and cueing from the teacher. How many zeros? How many decimal places? Write the decimal for these blocks.

ones	tenths	hundredth
1 .	3	4

Adapted from *Teaching Mathematics to Students with Learning Disabilities,* Bley & Thornton, p.296.

Teachers should continue to reinforce the decimals with models of the blocks (concrete level) until children can read and write the decimals (abstract level) for various examples. Children have difficulty understanding when a zero appears in the tenths place (Ex: 1.05). Omit using decimal examples like this at first, until children gain more understanding of place value.

How Is CRA Implemented?

CRA may be implemented at all grade levels individually, in small groups, or for the entire class. It can be used with children at the elementary or secondary level. When using CRA, the teacher should provide multiple opportunities for practice and demonstration to help students achieve mastery of the mathematical concept. The following guidelines are suggested when using manipulatives for accessibility to the mathematics concepts.

Guidelines for Using Manipulatives With Students With Disabilities

- Select manipulatives that are connected to the concept and to students' developmental level.
- Incorporate a variety of manipulatives for concept exploration and attainment.
- Provide verbal explanations and questions with demonstrations.
- Provide opportunities for student interaction and explanation.
- Encourage the use of manipulatives and strategies across settings.
- Program for transition from concrete to symbolic representation.

Maccini, P. & Gagnon, J. A. (2000, January). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children, 32*(5), 11.

A teacher can prompt students with questions at each stage of the process. If a student is solving a word problem, the teacher can read the problem aloud and summarize what the student completed as the student moves sequentially through the stages, using models, verbalization, drawings, and numerical representations to indicate each step in order. When implementing this strategy, a teacher practices good instruction by referring to concepts or activities in the different states. To reinforce concepts, instruction may be cyclic, not just a linear sequence of instructional tasks.

For Further Information

http://coe.jmu.edu/mathvidsr/disabilities.htm
This Web site provides instructional strategies in three categories: Teacher Instruction, Student Practice, and Evaluation. These strategies were chosen on the basis of learning research on students who are at risk of academic failure and students who are identified with learning problems, research on effective mathematics instruction for students with and without learning problems, and suggestions from an advisory committee made up of elementary and special education teachers

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