



PRESENTATION

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Monitoring Progress While Solving Problems

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Topic IMPROVING MATHEMATICAL PROBLEM SOLVING IN GRADES 4 THROUGH 8

Practice REFLECT AND DEBRIEF

- Highlights**
- » Dr. Sybilla Beckmann describes the importance of helping students reflect on the steps they are using while thinking through a problem.
 - » She provides an example of a problem that involves removing $\frac{2}{3}$ of an amount from a bowl, leaving $\frac{1}{4}$ of a cup. The challenge is to figure out how much was in the bowl before anything was removed.
 - » The first step in problem solving is restating the problem in students' own words, followed by identifying what they know about the problem and what they need to know.
 - » Dr. Beckmann illustrates why it is important for students to keep asking themselves whether their approaches to solutions make sense, including encouraging students to debate their approaches in light of revisiting the problem statement.
 - » She demonstrates scaffolding students' checking of reasoning and demonstrates bringing mathematical and algebraic notation into the stating, solving, and reasoning of the problem.

About the Interviewee

Sybilla Beckmann is Josiah Meigs Distinguished Teaching Professor of Mathematics at the University of Georgia. She has a Ph.D. in mathematics from the University of Pennsylvania and taught at Yale University as a J.W. Gibbs Instructor of Mathematics. Dr. Beckmann has done research in arithmetic geometry, but her current main interests are the mathematical education of teachers and mathematics content for students at all levels, but especially for pre-K through the middle grades. She developed several mathematics content courses for prospective elementary school teachers at the University of Georgia and wrote a book for such courses, *Mathematics for Elementary Teachers*, published by Addison-Wesley, now in a third edition. She is interested in helping college faculty learn to teach mathematics content courses for elementary and middle grades teachers, and she works with graduate students and postdoctoral fellows toward that end. As part of this effort, Dr. Beckmann directs the Mathematicians Educating Future Teachers component of the University of Georgia Mathematics Department's VIGRE II grant.

Dr. Beckmann was a member of the writing team of the National Council of Teachers of Mathematics' *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*; was a member of the Committee on Early Childhood Mathematics of the National Research Council and co-author of its report, *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*; has worked on the development of several state mathematics standards; and was a member of the mathematics writing team for the Common Core State Standards Initiative. Several years ago Dr. Beckmann taught an average sixth-grade mathematics class every day at a local public school in order to better understand school mathematics teaching.

Full Transcript



Slide 1: Welcome

Welcome to Monitoring Progress While Solving Problems.



Slide 2: Introducing Sybilla Beckmann

I am Sybilla Beckmann, Josiah Meigs Distinguished Teaching Professor at the University of Georgia. I was a member of the panel for the Problem Solving guide as well as a member of the panel for the Response to Intervention guide.



Slide 3: Reflecting is thinking deeper

So what does monitoring and reflecting on problem solving look like in the classroom? Well, overall, the key thing is that we want students to be thinking about their thinking and thinking about problem solving. What we want is something that is more focused on making sense of problems and of solution methods and something that is more reflective and goes deeper into the problem-solving process.



Slide 4: Problem statement

Here is a problem about some clay in a bowl:

There was some clay in a bowl. After I took out $\frac{2}{3}$ of it, there was $\frac{1}{4}$ of a cup of clay left. How much clay was in the bowl at first?

So let's think about understanding this problem. When we read a problem like this, it's very tempting for students to simply do something with those numbers— $\frac{2}{3}$ and $\frac{1}{4}$. But it's very important, a very key first step is to really understand what that problem is asking and what is it about.



Slide 5: Restate the problem

Often it's good to just restate that problem in your own words. I can restate this as, "I had some clay, I took some clay out, and what do I know? I know how much is left." So that's the overall big picture of what's happening in the problem.



Slide 6: Questions to ask about problem

When students solve a problem like this, it's good for them to be asking themselves and each other questions about what they want to know and what they already know from the problem statement. We know that's what we're looking for in this problem. We want to know how much clay did we start with in the bowl. I know I took some out, I know that amount that I took out was $\frac{2}{3}$ of it, and I know that when I am done taking that clay out, I have $\frac{1}{4}$ of a cup of clay left.



Slide 7: Initial solution attempts

So at this point, students could start thinking about how might they start to solve this problem. And a student might right away think of subtraction because something is being removed from the bowl. "Maybe I can subtract $\frac{1}{4}$ from $\frac{2}{3}$." And at this point, students should be asking themselves and each other if that makes sense: "Well, that doesn't quite fit because, look, we took out $\frac{2}{3}$ of the clay. That would be the amount that's taken out. It shouldn't be $\frac{2}{3}$ minus $\frac{1}{4}$. That really just doesn't fit."



Slide 8: Debating initial approaches

One student might think, "Well, wait a second. If I took out $\frac{2}{3}$, how can I have $\frac{1}{4}$ left because if you take out $\frac{2}{3}$, shouldn't you have $\frac{1}{3}$ left? Hmm..." So this might be the kind of point in problem solving where students debate with each other and try to come to some agreement or some consensus about what is stated in the problem and what is it that we are looking for.



Slide 9: Rereading the problem

So let's go back and really try to understand the problem one more time. And if students go back and read it they see, "After I took out $\frac{2}{3}$ of it, there was $\frac{1}{4}$ of a cup of clay left." The $\frac{2}{3}$ and the $\frac{1}{4}$ refer to different things. They are not both referring to cups of clay. The $\frac{2}{3}$ is referring to the amount of clay that was in the bowl, whereas the $\frac{1}{4}$ is referring to a cup of clay.



Slide 10: Clarifying problem terms

So after $\frac{2}{3}$ of the clay is removed, it is the case that $\frac{1}{3}$ of the clay that was originally in the bowl is left. We're given in the problem that this $\frac{1}{3}$ of the clay that's left in the bowl is $\frac{1}{4}$ of a cup of clay. And this means that what was originally in the bowl is three times as much as this $\frac{1}{4}$ of a cup, which means that what was originally in the bowl must be three sets of $\frac{1}{4}$ of a cup, or three pieces, each of which is a fourth of a cup, which is what we mean by $\frac{3}{4}$ of a cup.



Slide 11: Checking the reasoning—adding notation

We have reasoned through that the initial amount in the bowl was $\frac{3}{4}$ of a cup. Is it true that if we take out $\frac{2}{3}$ of that then we'll have a fourth of a cup left? $\frac{1}{3}$ of the initial amount of clay is equal to $\frac{1}{4}$ of a cup, and that gives us a multiplication sentence. $\frac{1}{3}$ times the initial amount of clay is equal to $\frac{1}{4}$, which could be solved by dividing $\frac{1}{4}$ by $\frac{1}{3}$, and that again gives us the $\frac{3}{4}$ of a cup. So this would be a way to reflect on the problem and then to see it in a new light in terms of a multiplication equation.



Slide 12: Prompts to reconsider

We can see that if students simply do the first thing that pops into their mind, it may not be correct. And unless students have some mechanism by which they are prompted to rethink what they have done or to reflect on whether it makes sense what they've just done, they may simply rush headlong to a solution and not ever reconsider what it was that they did.



Slide 13: Learn more

To learn more about Monitoring Progress While Solving Problems, please explore the additional resources on the Doing What Works website.

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